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THE EFFECT OF PROBLEM BASED LEARNING MODEL ON IMPROVING MATHEMATICAL CRITICAL THINKING ABILITY AND STUDENT LEARNING INDEPENDENCE IN MAN 1 MEDAN

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Abstract

This study investigated the effectiveness of Problem Based Learning (PBL) compared to the Traditional Learning Model in mathematics education, focusing on two key aspects: students' mathematical critical thinking skills and learning independence. The research examined whether PBL yielded better results than traditional methods in enhancing these capabilities, while also analyzing potential interactions between learning approaches and students' initial mathematical abilities. The study was conducted with a sample of 60 tenth-grade students from MAN 1 Medan, drawn from classes X-9 and X-6, who were divided into experimental and control groups. Using mathematical critical thinking skills assessments and a learning independence scale, data were analyzed through two-way ANOVA. The results demonstrated that the PBL model was significantly more effective than the Traditional Learning Model in improving both students' mathematical critical thinking skills and learning independence. However, no significant interactions were found between learning approaches and students' initial mathematical abilities in either the development of critical thinking skills or learning independence. These findings strongly suggest that Problem Based Learning should be considered as an effective instructional strategy for educators seeking to enhance their students' mathematical critical thinking skills and foster greater learning independence in the classroom.

Keywords: Problem-Based Learning, Critical Thinking, Learning Independence.

Abstrak

Penelitian ini menyelidiki efektivitas Problem Based Learning (PBL) dibandingkan dengan Model Pembelajaran Biasa dalam pendidikan matematika, berfokus pada dua aspek utama: keterampilan berpikir kritis matematika siswa dan kemandirian belajar. Penelitian ini mengkaji apakah PBL menghasilkan hasil yang lebih baik daripada metode pembelajaran biasa dalam meningkatkan kemampuan berpikir kritis dan kemandirian belajar siswa, sambil juga menganalisis interaksi potensial antara pendekatan pembelajaran dan kemampuan matematika awal siswa. Studi ini dilakukan dengan sampel 60 siswa kelas X dari MAN 1 Medan, yang diambil dari kelas X-9 dan X-6, yang dibagi menjadi kelompok eksperimen dan kontrol. Dengan menggunakan asesmen keterampilan berpikir kritis matematika dan skala kemandirian belajar, data dianalisis melalui ANOVA dua arah. Hasilnya menunjukkan bahwa model PBL secara signifikan lebih efektif dari pada Model Pembelajaran Biasa dalam

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meningkatkan baik keterampilan berpikir kritis matematika siswa maupun kemandirian belajar mereka. Namun, tidak ditemukan interaksi yang signifikan antara pendekatan pembelajaran dan kemampuan matematika awal siswa, baik dalam pengembangan keterampilan berpikir kritis maupun kemandirian belajar. Temuan ini sangat menyarankan agar PBL dipertimbangkan sebagai strategi instruksional yang efektif bagi para pendidik yang berusaha meningkatkan keterampilan berpikir kritis matematika siswa dan menumbuhkan kemandirian belajar yang lebih besar di kelas.

Kata Kunci: Problem Based Learning,Berpikir Kritis, Kemandirian Belajar.

INTRODUCTION

Mathematics occupies a vital role within the educational system, as evidenced by the significant instructional time allocated to mathematics courses across all levels of schooling, from elementary to secondary to higher education. This prominence reflects the fundamental importance of mathematics in equipping students with the quantitative reasoning skills, analytical capabilities, and problem-solving expertise required to navigate an increasingly technological and data-driven world. Mastery of mathematical concepts and competencies is seen as essential not only for academic success, but also for thriving in future careers and civic life. By devoting substantial curricular focus to mathematics, educational institutions recognize the critical function this subject plays in developing the cognitive abilities, analytical mindset, and problem-solving prowess that students will need to excel in the modern, interconnected, and technologically-advanced society of the future. The sustained emphasis on mathematics education at all grade levels underscores its indispensable role in preparing learners to face the evolving challenges and complex problems that will arise in the era of globalization and rapid scientific and technological progress.

Reviewing the mathematics learning objectives outlined in the educational standards, the mathematics learning process must be designed to help and provide ample opportunities for students to develop a comprehensive understanding of mathematical concepts, reasoning capabilities, communication skills, problem-solving expertise, and positive attitudes towards learning mathematics (Ministry of Education, 2006: 139). As stated in the content standards for elementary and secondary education units of mathematics subjects, mathematics courses need to be provided to all students starting from the elementary level in order to equip them with logical, analytical, systematic, critical, and creative thinking skills, as well as the ability to collaborate effectively. It is hoped that through effective mathematics education, students can cultivate these essential thinking skills, particularly those that lead to the development of strong mathematical critical thinking abilities. The emphasis on mathematics learning across all grade levels reflects the recognized importance of nurturing these cognitive capabilities, which are crucial for student success not only within the academic setting, but also in navigating the challenges of the modern, interconnected world.

Mathematical critical thinking forms the foundation of the cognitive process required to analyze arguments and generate ideas in a logical manner (Noer, 2009:474). This form of critical thinking not only leads to drawing conclusions about what to believe, but also informs the actions that should be taken. As such, the ability to think critically in a mathematical context is not solely about finding answers, but more importantly, involves questioning the information provided and exploring alternative perspectives. The development of mathematical critical thinking skills enables students to move beyond simply recalling facts or memorizing procedures, equipping them with the analytical capabilities needed to deeply examine problems, assess the validity of claims, and synthesize creative solutions - abilities that are vital for success not just in mathematics, but across numerous academic and real-world domains requiring rigorous, logical reasoning. By emphasizing the cultivation of mathematical critical thinking skills, educators can foster the kind of analytical mindset and problem-solving prowess that will prepare students to navigate the increasingly complex challenges of the modern world, enabling them to become independent, innovative, and discerning thinkers, able to make well-informed decisions that drive positive change.

According to Susanto (2013:121), mathematical critical thinking involves the cognitive activity of contemplating ideas or concepts related to a given problem, while Ismaimuza (2010) defines it as a thought process aimed at making reasonable decisions about what is believed to be the truth. From these expert perspectives, mathematical critical thinking can be understood as an effective, higher-order thinking skill that enables individuals to make, evaluate, and determine appropriate beliefs and actions.

In fact, mathematical critical thinking serves as the foundation for three other advanced thinking patterns - creative, logical, and reflective thinking. This means that mastery of critical thinking in a mathematical context is a prerequisite for developing these other crucial cognitive abilities. Consequently, the cultivation of mathematical critical thinking skills is of paramount importance for students, as these capabilities allow them to think rationally, assess alternatives, and make the best-informed decisions.

By honing their mathematical critical thinking skills, students gain the analytical prowess needed to deeply examine problems, assess the validity of claims, and synthesize creative solutions. This type of rigorous, logical reasoning is vital not only for success in mathematics, but across numerous academic and real-world domains that require independent, innovative, and discerning thought. When educational institutions emphasize the development of mathematical critical thinking, they are effectively preparing students to navigate the increasingly complex challenges of the modern world, empowering them to become well-informed decision-makers who drive positive change.

Moreover, instilling the habit of mathematical critical thinking for students is crucial, as it enables them to observe and analyze various problems that occur in their daily lives (Somakim, 2011:43). By developing the ability to think critically in a mathematical context, students become more resilient in facing and addressing real-world challenges. This critical thinking capacity allows them to apply the knowledge gained in school to solve problems appropriately in diverse, authentic situations.

However, based on the results of observations conducted by researchers on Friday, January 12, 2024 at MAN 1 Medan, where 30 students were given a mathematical critical thinking assessment, it was found that the students at this school are still severely lacking in these essential skills. The findings from this initial assessment reveal that cultivating mathematical critical thinking abilities among MAN 1 Medan students remains an area that requires significant improvement and attention from educators.

The development of mathematical critical thinking is not only important for students' academic success, but also prepares them to navigate the complexities of the modern world. By honing these higher-order thinking skills, students gain the analytical capabilities needed to deeply examine problems, assess the validity of claims, and synthesize creative solutions - abilities that are vital across numerous professional and civic domains. As such, educational institutions must prioritize the systematic fostering of mathematical critical thinking to empower students to become independent, innovative, and discerning thinkers, able to make well-informed decisions that drive positive change.

Table 1. Students' level of mathematical critical thinking ability

Indicators of Students' Mathematical Critical Thinking Ability	Students who can answer	Percentage	Average Student KBKM
Identify	21	70%	22,5% (Very Lacking)
Generalize	6	20%	
Analyze	0	0%	
Troubleshooting	0	0%	

After carefully analyzing the results of the initial assessment of students' critical thinking abilities, the data displayed in the table above reveals that the average score attained by the students is 22.5, which falls within the "Very Lacking" category according to the evaluation criteria established by Sudijono (2010). This assessment framework defines five distinct performance levels: "Very Lacking, Lacking, Sufficient, Good, and Very Good".

The findings from this initial diagnostic evaluation clearly indicate that the mathematical critical thinking skills of the students require significant improvement and attention. When educational institutions are able to accurately assess the current state of learners' critical thinking capabilities, they can then implement targeted instructional strategies and interventions to systematically develop these essential higher-order thinking skills. By cultivating students' mathematical critical thinking abilities, educational institutions equip learners with the analytical prowess needed to deeply examine problems, assess the validity of claims, and synthesize creative solutions. These cognitive skills are vital not only for academic success, but also for navigating the increasingly complex challenges of the modern world. As such, the priority placed on nurturing mathematical critical thinking should be a key focus for educators seeking to empower students to become independent, innovative, and discerning thinkers, capable of making well-informed decisions that drive positive change.

Table 2. Critical Thinking Ability Initial Score Criteria Interval

It	Value Interval	Categories Assessment
1	$0 \leq KBKM < 45$	Very Lacking Lacking
2	$45 \leq KBKM < 65$	Lacking
3	$65 \leq KBKM < 75$	Sufficient
4	$75 \leq KBKM < 90$	Good
5	$90 \leq KBKM < 100$	Very Good

In addition to mathematical critical thinking skills, other important affective aspects involved in the learning process include learning independence. Learning independence is a skill wherein the individual is encouraged, controlled, and assessed by themselves during the learning activities (Lilik et al., 2013: 64). This means that students manage their own learning by activating their cognitive, affective, and behavioral capacities to achieve the desired learning goals (Sumarmo, 2004:5).

The need for developing learning independence among individuals studying mathematics is further supported by several research findings. Studies have shown that students with high levels of learning independence tend to learn more effectively, as they are better able to monitor, evaluate, and manage their own learning processes. These self-directed learners also tend to complete their tasks in a timelier manner, organize their study and time efficiently, and obtain higher scores in mathematics (Hargis in Sumarmo, 2004:5). Additionally, according to Fauzi (2011:111), the importance of learning independence in mathematics education is driven by the demands of the curriculum, which require students to be able to tackle increasingly complex problems both inside and outside the classroom. Cultivating learning independence also helps reduce students' dependence on others in their daily lives. In this way, mathematics education can empower students to become more autonomous, self-regulated learners. Taken together, the development of both mathematical critical thinking skills and learning independence are crucial for preparing students to thrive in the modern, complex world. By simultaneously nurturing these essential cognitive and affective capabilities, educational institutions can equip learners with the analytical, problem-solving, and self-directed learning abilities needed to make well-informed decisions and drive positive change.

Currently, students have not fully developed learning independence, as they still view teachers as their sole source of knowledge. This mindset hinders students' ability development and creates dependency on others, particularly teachers. However, as Yamin (2008: 204-205) notes, knowledge can be acquired through various sources, locations, facilities, and the surrounding environment, including libraries, laboratories, and the internet.

This situation is evident at MAN 1 Medan. Based on the researcher's observations and an interview with a mathematics teacher at the school, most students exhibit a strong dependence on their teachers for learning. Students tend to be passive, merely receiving information and instructions from teachers. They rarely ask questions about the presented material and often display uncertainty when solving problems due to a lack of confidence in their own abilities, indicating low levels of learning independence.

Students' low critical thinking skills in mathematics and lack of learning independence stem from several factors, including teachers' instructional approaches. Many teachers have not effectively selected and implemented appropriate learning models to convey educational materials and objectives. In mathematics education, most teachers still rely on conventional teaching methods. This is supported by a research survey conducted by Triyono, et al. (2023), which found that numerous mathematics teachers across all educational levels—elementary, junior high, and high school—continue to use traditional teacher-centered learning approaches.

To address these challenges, teachers need to select appropriate learning models that can enhance students' mathematical critical thinking skills and learning independence. Problem Based Learning (PBL) is considered one of the suitable approaches to achieve these objectives. According to Cahyo (2013: 283), Problem Based Learning is characterized by student-centered learning through the introduction of problems at the beginning of lessons, serving as a starting point for acquiring and integrating new knowledge. This allows students to solve problems by utilizing their existing knowledge to construct new understanding. Similarly, Soedjadi (2000: 99) states that Problem Based Learning begins with complex real-world problems, which are then broken down into interrelated simple concepts.

Problem Based Learning was selected not only for its potential to enhance students' critical thinking skills but also for its student-centered nature and emphasis on learning independence (Riyanto, 2010: 291). This is further supported by Eggen and Kauchak (2012: 307), who assert that Problem Based Learning uses problems as a focal point to develop students' problem-solving abilities, subject matter understanding, and learning independence.

Teachers must also consider students' initial mathematical abilities, particularly their initial mathematical critical thinking skills, which are the focus of this study. According to Ismaimuza (2010: 3), students' initial mathematical abilities are the skills they possess before beginning the mathematics learning process. These initial abilities vary among students and can be categorized into high, medium, and low levels based on their mastery. As Uno (2008:

61) explains, while some students may only be at the introductory stage of mathematical understanding, others might have already reached the ready-to-use stage for the same content. Therefore, it is crucial for teachers, as classroom instructional designers, to carefully consider these varying levels of initial mathematical ability when planning their lessons.

Therefore, a teacher must assess students' initial mathematical abilities to minimize potential difficulties students may face in understanding the new material to be taught. This enables students to progressively improve their mathematical capabilities. Additionally, by understanding students' varied initial mathematical abilities, teachers can select appropriate learning models for classroom implementation, ensuring that the learning process remains effective.

Based on the preceding discussion, it is necessary to conduct research examining the implementation of Problem Based Learning, which is expected to enhance students' mathematical critical thinking skills and learning independence. Therefore, this study addresses the following research questions: (1) Whether the influence of *the Problem Based Learning* learning model is better than the Ordinary Learning Model on improving students' mathematical critical thinking skills in MAN 1 Medan (2) Is there an interaction between the learning model and students' initial mathematical ability (KAM) on improving students' mathematical critical thinking skills (3) Whether the influence of *the Problem Based Learning* learning model is better than the Ordinary Learning Model on increasing student learning independence in MAN 1 Medan (4) Whether there is an interaction between the learning model and students' initial mathematical ability (KAM) on increasing student learning independence.

METHODS

This research employs a quasi-experimental design, which utilizes a control group but cannot fully control external variables that may influence the experiment's implementation. The study groups samples based on pre-existing classes rather than forming new ones, making it a quasi-experimental method that works with established class structures.

The purpose of this study is to examine two key aspects: first, to determine differences in the improvement of mathematical critical thinking abilities and learning independence between students exposed to Problem Based Learning versus those receiving regular instruction; and second, to investigate the interaction between the learning approach and students' initial mathematical abilities in enhancing mathematical critical thinking skills and learning independence.

Data analysis is conducted through inferential analysis methods. This inferential analysis serves to evaluate the improvement in students' mathematical critical thinking abilities and learning independence, as well as to examine the interaction between learning approaches and students' initial mathematical abilities through two-way ANOVA, focusing on both mathematical critical thinking and learning independence.

This research is conducted at MAN 1 Medan, located at Jl. William Iskandar No.7B, Bantan Tim., Medan Tembung, Medan City, North Sumatra 20222. The study takes place during the Odd Semester of the 2024/2025 Academic Year. The implementation is scheduled for July 2024, comprising 4 meetings each for both the experimental and control classes.

At the beginning of the study, both groups are administered a pre-test using descriptive questions and learning independence scale sheets to measure initial conditions. Subsequently, the experimental group receives treatment in the form of Problem Based Learning, while the control group follows regular instruction. Upon completion of the treatment period, both groups are given a post-test and learning independence scale sheet. As outlined by Arikunto (2009:210), this study employs the following research design:

Table 3. Research Design

Group	Class	Pretest	Treatment	Postes
Experimental Group	X-9	0_1	X1	0_2
Control Group	X-6	0_1	X2	0_2

The research employs both test and non-test instruments. The test instruments include assessments of students' initial mathematical ability and mathematical critical thinking skills, while non-test instruments comprise learning independence scale sheets and observation sheets for learning activities. All instruments have been validated for reliability, difficulty level, and discrimination power through expert review and non-respondent testing. Based on the data collection methods employed, two types of data are gathered: quantitative and qualitative. Quantitative data is obtained through three sources: initial mathematical ability tests, mathematical critical thinking ability tests, and learning independence scales completed by students. In addition to quantitative analysis, qualitative analysis will be conducted on observations of teacher learning activities in the experimental class, which implements Problem Based Learning. The quantitative data analysis is performed for each pair of data groups according to the research questions. The quantitative data undergoes analysis through three stages:

1. The first stage: Data was collected from mathematical critical thinking ability scores and learning independence assessments. The data processing began by calculating the normalized gain between pretest and posttest scores. This stage revealed improvements in students' critical thinking skills and learning independence before and after receiving either problem-based learning or conventional instruction. The normalized gain (Ng) was calculated using Hake's formula.:

$$Ng = \frac{skor\ \ postes - skor\ \ pretes}{skor\ \ maksimal\ ideal - skor\ \ pretes} \qquad \text{(Hake, 1998 : 65)}$$

Table 4. Normalized Gain Score Criteria

Gain Score	Interpretation
$g < 0.3$	Low
$0.3 < g \leq 0.7$	Keep
$g \geq 0,7$	Tall

2. The second stage: The N-gain results were analyzed using the following statistical procedures:

a. Normality Test

In univariate analysis, data must come from a normally distributed population. The normality test determines whether data distribution follows or approximates a normal distribution. Each variable was tested for normality individually, as normally distributed variables collectively indicate overall normal distribution. This study utilized SPSS version 26.0 software.

For individual testing of mathematical critical thinking ability and student learning independence scores, the Kolmogorov-Smirnov test was performed using SPSS Statistics 26.0. The testing procedure was as follows:

a) Determine the α value (the α value used in this study is 0.05).
b) Processing the data obtained using *SPSS 26.0 statistics software* is by clicking analyze, then selecting descriptive statistics, then selecting explore, then entering the gain value in the

dependent box and learning in the factor list box, then clicking plots and selecting normality plots with test and finally clicking ok.

c) If in the sig. The value is more than α = 0.05 then H0 is accepted.

The hypotheses to be tested are as follows:

$H_0 : {\sigma_1}^2 = {\sigma_2}^2$ This means that the sample comes from a normally distributed population

$H_1 : {\sigma_1}^2 \neq {\sigma_2}^2$ This means that the sample comes from a population that is not normally distributed

b. Homogeneity Test

In addition to the normality test, univariate analysis requires a homogeneity test. This test examines the variance similarity between the experimental and control groups to determine whether the variances of both groups are equal or different.The steps to conduct the test are:

1) Define the statistical hypothesis for the similarity of univariate variance matrices

This revised version

$H0 : \sigma1^2 = \sigma2^2$

$Ha : \sigma1^2 \neq \sigma2^2$

Information:

$\sigma1^2$ = Variance of Problem *Based Learning Score*

$\sigma2^2$ = Variance of the typical learning score

2) Set the significance level α (0.05 for this study)
3) Process the data obtained by using *SPSS 26 statistics software* click analyze, then select descriptive statistic, then select explore, then enter the gain value in the dependent box and learning in the factor list box, then click plots and on the Levene test spread option click untrasformed with test and finally click ok.
4) Accept H_0 if the significance value (sig) is greater than α = 0.05. The Levene Test is used to assess homogeneity..

c. Anava Two Ways

Two-way ANOVA is a research data analysis technique that examines factorial designs with two factors. According to Arikunto (2009:424), this statistical method is used in research where two variables serve as the basis for reviewing scores of a dependent variable. In this context, two-way ANOVA is used to test all research questions outlined in the background.

Testing criteria:

- If F0 < Ft at the selected significance level with appropriate degrees of freedom, then H0 is rejected
- If F0 > Ft at the selected significance level with appropriate degrees of freedom, then H0 is accepted

In addition to manual calculation methods, two-way ANOVA can be performed using SPSS 23.0 Statistical software. The testing procedure is as follows:

1) Determine the α value (the α value used in this study is 0.05).
2) Process the data obtained using *SPSS 26.0 statistics software* by clicking analyze, selecting the general linear model and then clicking univariate. Next, enter the gain value into the dependent variable box, and enter the learning and KAM into the fixed factors box. Then click on plots and enter KAM on the horizontal axis box and learning to the separate line box then click add and continue. Next, click on post hoc and enter KAM in the post hoc for box and check the tukey and bonferroni on the equal variances assumed option. After that, click the option box and enter the value of KAM, learning, KAM*learning in the means for display box and check descriptive and homogeneity on the display option, then click continue and ok.
3) If in the sig. the value is more than α = 0.05 then H0 is accepted.

d. Problem Formulation Test

$$Yijk = + \mu\alpha_i + \ + \beta_j + (\alpha\beta)_{ij}\varepsilon_{Ijk.} \ ; i=1,2 \ ; j=1,2 \ ; k = 1,2,... 30 \quad (1)$$

The statistical model to answer the formulation of the above research problem used is a two-way anova.

RESULTS AND DISCUSSION

This sub-chapter provides a comprehensive analysis of students' Initial Mathematical Ability (KAM), mathematical critical thinking skills, and learning independence. The analysis begins with an examination of pretest and posttest results for both mathematical critical thinking skills and learning independence, followed by their respective N-Gain calculations. Furthermore, this section presents various statistical analyses, including normality tests, homogeneity tests, mean difference tests, and two-way ANOVA, to evaluate the collected data. Through these analyses, we aim to provide a thorough understanding of the relationships and changes in students' mathematical abilities and learning independence throughout the study period.

1. Description of the results of the student's initial mathematics ability

Students' Initial Mathematical Ability (KAM) is assessed through a test designed to classify students into three proficiency levels: high, medium, and low. This assessment utilizes five descriptive questions derived from National Examination (UN) materials. To analyze students' KAM performance, mean scores and standard deviations were calculated, as presented in the following table:

Table 5. Description of the results of the Student's Initial Mathematics Ability Test

Class	N	Xmin	Xmax	Average	SD
Experimental Classes	30	35	75	57,13	11,85
Control Classes	30	30	78	56,9	11,77

The subsequent analysis includes tests for normality, homogeneity, and mean differences, with results presented as follows:

Table 6. Results of the Normality Test of Students' Initial Mathematics Ability Scores

Tests of Normality

Class		Kolmogorov-Smirnova			Shapiro-Wilk		
		Statistics	Df	Sig.	Statistics	Df	Sig.
THURS	experiment	.148	30	.093	.936	30	.072
	control	.130	30	.200*	.963	30	.364

* This is a lower bound of the true significance.
a Lilliefors Significance Correction

Based on Table 6, the Kolmogorov-Smirnov test shows significance values of 0.093 for the experimental class and 0.200 for the control class. Since both values exceed the significance level of 0.05, we can conclude that the data from both classes follows a normal distribution.

Table 7. Results of the Homogeneity Test of Students' Initial Mathematics Ability

Test of Homogeneity of Variance

		Levene Statistic	df1	DF2	Sig.
KAM Results	Based on Mean	.450	1	58	.505
	Based on Median	.315	1	58	.577
	Based on Median and with adjusted df	.315	1	57.997	.577
	Based on trimmed mean	.415	1	58	.522

The Levene test results from SPSS analysis reveal a significance value of 0.505 for KAM, which is greater than α = 0.05. This indicates that the initial ability test data from both control and experimental classes are homogeneous.

Table 8. Results of the Average Difference Test of Student KAM Results

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference Lower	95% Confidence Interval of the Difference Upper
THURS	Equal variances assumed	.450	.505	.305	58	.762	.967	3.174	-5.387	7.320
	Equal variances not assumed			.305	57.855	.762	.967	3.174	-5.387	7.320

Analysis of Table 8 shows a significance value of 0.762, which exceeds the 0.05 threshold, leading to the acceptance of H0. This confirms that there is no significant difference in ability variance between the experimental and control groups. Therefore, we can conclude that both groups demonstrate equivalent variances.

2. Description of the results of N-Gain Students' Mathematical Critical Thinking Ability

Before looking at the improvement, we will first present a pre-test and post-test table for each sample class:

Table 9. Results of the Mathematics Critical Thinking Ability Test in the Experimental Class

Experimental Classes	Maximum Score	Xm_{in}	*Xmax*	Average	SD	Percentage
Pretest	100	27	57	41,4	6,72	41,4%
Postes	100	41	99	71,23	16,39	71,23 %

Table 10. Results of the Mathematical Critical Thinking Ability Test in the Control Class

Control Classes	Maximum Score	Xm_{in}	*Xmax*	Average	SD	Percentage
Pretest	100	28	57	41,66	7,26	41,66%
Postes	100	38	90	65,53	15,97	65,53 %

Table 11. N-Gain Results of Students' Mathematical Critical Thinking Skills

Class	Xm_{in}	*Xmax*	Average	SD
Experiment	0,032	0,978	0,527	0,254
Control	0,005	0,95	0,446	0,243

Furthermore, a normality and homegency test was carried out on the N-gain students' mathematical critical thinking skills, the results were as follows

Table 12. Results of N-Gain Normality Test for Mathematical Critical Thinking Ability in Experimental Class and Control Class

Tests of Normality

	Kolmogorov-Smirnova			Shapiro-Wilk		
	Statistics	Df	Sig.	Statistics	Df	Sig.
EXPERIMENT	.106	30	.200*	.976	30	.698
CONTROL	.102	30	.200*	.948	30	.153

*. This is a lower bound of the true significance.
a. Lilliefors Significance Correction

Based on the Kolmogorov-Smirnov test results, both the experimental and control classes showed a significance value of 0.200. Since this value exceeds the alpha level of 0.05, we can conclude that the N-gain data from both classes follow a normal distribution.

Table 13. Results of N-Gain Homogeneity Test for Mathematical Critical Thinking Ability in Experimental Class and Control Class

Test of Homogeneity of Variance

		Levene Statistic	df1	DF2	Sig.
Gain Thinking Critical	Based on Mean	2.072	1	58	.155
	Based on Median	1.689	1	58	.199
	Based on Median and with adjusted df	1.689	1	52.930	.199
	Based on trimmed mean	2.032	1	58	.159

Based on the Levene's test conducted using SPSS 26, the significance value obtained was 0.155, which is greater than α = 0.05. This leads to the acceptance of the null hypothesis, indicating that all populations have equal (homogeneous) variances. Therefore, we can conclude that the N-gain data of mathematical critical thinking ability demonstrates homogeneous variance across all groups.

3. Description of N-Gain Results of Student Learning Independence

Before looking at the increase in learning independence scores, a table of the starting and ending scores of the learning independence scale for each sample class will first be presented.

Table 14. Results of Student Learning Independence Scale in Experimental Classes

Experimental Classes	Maximum Score	Xm_{in}	*Xmax*	Average	SD	Percentage
Pretest	125	78	99	87,36	4,722	69,88 %
Postes	125	108	120	113,9	2,86	91,12 %

Table 15. Results of Student Learning Independence Scale in the Control Class

Experimental Classes	Maximum Score	*Xmin*	*Xmax*	Average	SD	Percentage
Pretest	125	76	97	86,8	6,029	68,28 %
Postes	125	87	123	103,96	9,50	83,17 %

Table 16. N-Gain Results of Student Learning Independence in Both Sample Classes

Class	*Xmin*	*Xmax*	Average	SD
Experiment	0,384	0,868	0,697	0,10
Control	0,055	0,956	0,446	0,243

Furthermore, normality, homegenity and average differences were tested, the results were as follows

Table17. Results of the N-Gain Normality Test for Classroom Learning Independence Experiments and Control Classes(*Tests of Normality*)

Tests of Normality

Class		Kolmogorov-Smirnova			Shapiro-Wilk		
		Statistics	Df	Sig.	Statistics	Df	Sig.
N-Gain Student Learning Independence	EXPERIMENT	.106	30	.200*	.964	30	.386
	CONTROL	.090	30	.200*	.945	30	.124

From Based on the Kolmogorov-Smirnov test results, both the experimental and control classes yielded a significance value of 0.200. Since this value is greater than the alpha level of 0.05, we can conclude that the N-gain data from both classes follow a normal distribution.

Table 18. Results of N-Gain Homogeneity Test for Learning Independence in Experimental Classes and Control Classes

Test of Homogeneity of Variance

		Levene Statistic	df1	DF2	Sig.
N-Gain Independence Learn	Based on Mean	1.075	1	58	.304
	Based on Median	1.037	1	58	.313
	Based on Median and with adjusted df	1.037	1	57.387	.313
	Based on trimmed mean	1.053	1	58	.309

The results of the Levene test conducted using SPSS 26 showed a significance value of 0.304, which is greater than α (0.05). Therefore, the null hypothesis is accepted, indicating that all populations have equal or homogeneous variance.

4. **Two-Way Anova Test on Problem Formulation**

a) **Measuring the effect of *the Problem Based Learning* model better than the Ordinary Learning Model on improving students' mathematical critical thinking skills**

The decision rule for hypothesis testing states that if the significance value (sig) is greater than or equal to 0.05, we accept the null hypothesis (H0); if it is less than 0.05, we reject it. This analysis was performed using SPSS version 26.

Table 19. Summary of the ANAVA Two-Path Thinking Ability Gain Test Critical Mathematics

Tests of Between-Subjects Effects

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	1.972a	5	.394	37.440	.000
Intercept	6.540	1	6.540	620.855	.000
LEARNING MODEL	.043	1	.043	4.096	.000
KAM	1.325	2	.662	62.871	.000
LEARNING MODEL * KAM	.001	2	.001	.487	.953
Error	.569	54	.011		
Total	16.800	60			
Corrected Total	2.541	59			

Based on the analysis shown in Table 19, the significance value for the learning model was found to be 0.000, which is less than 0.05. Therefore, we reject the null hypothesis (H0) and accept the alternative hypothesis (Ha). This finding demonstrates that the Problem-Based Learning (PBL) model is more effective than the Ordinary Learning Model in developing students' mathematical critical thinking skills. The results confirm that there is a significant difference between these two teaching approaches in terms of enhancing students' mathematical critical thinking abilities.

b) **Measuring whether there is no interaction between the learning model and students' initial mathematical ability on the improvement of students' mathematical critical thinking skills.**

According to the test criteria, the alternative hypothesis (Ha) is accepted if the significance value (sig) exceeds 0.05; otherwise, it is rejected. Analysis of Table 4.20 reveals that the interaction between LEARNING MODEL and Initial Mathematical Ability (KAM) produces a significance value of 0.953 (sig. > 0.05). Consequently, the null hypothesis (H0) is rejected while the alternative hypothesis (Ha) is accepted, indicating that there is no significant interaction between the learning model employed and students' initial mathematical abilities in terms of improving students' mathematical critical thinking skills. This statistical relationship is visually represented in Figure 1.

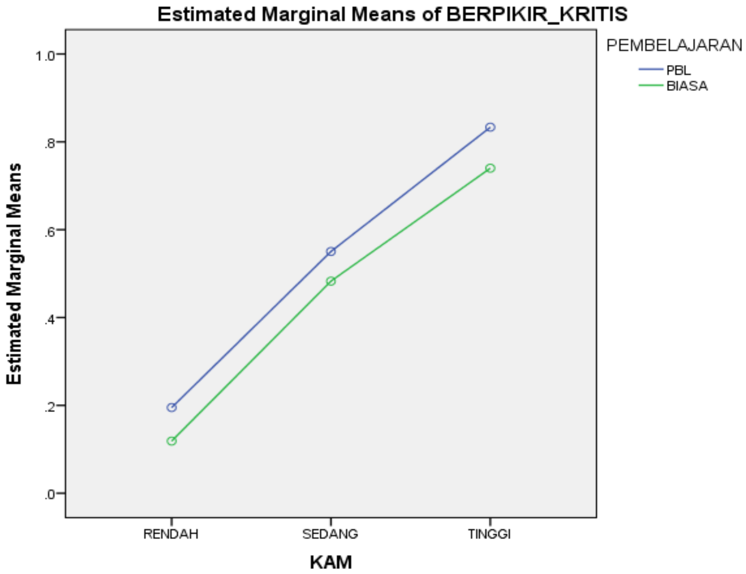

Figure 1. Interaction between the Learning Model and Students' KAM Against Improving Students' Mathematical Critical Thinking Skills

As illustrated in Figure 1, the experimental class line (Problem Based Learning) maintains a parallel relationship without intersecting the control class line (Regular Learning), indicating no interaction between the learning model and Initial Mathematical Ability (KAM - high, medium, low) in improving students' mathematical critical thinking skills. This demonstrates that there is no combined effect between the learning model and students' KAM on the development of mathematical critical thinking abilities. This outcome can be attributed to the inherent differences in students' initial capabilities - students enter with varying ability levels (high, medium, and low). Students with high initial ability consistently achieve better results regardless of the teaching methodology employed, while students with medium or low abilities may not necessarily show improved learning outcomes despite the learning model used.

c) Measuring the effect of the Problem *Based Learning* learning model better than the Regular Learning Model on increasing student learning independence

The statistical test criteria specify that the null hypothesis (H0) is accepted if the significance value (sig) is greater than or equal to 0.05; otherwise, it is rejected. The statistical analysis was performed using SPSS version 26 to determine these results.

Table 20. Summary of the ANAVA Test Two Paths to Gain Learning Independence

Tests of Between-Subjects Effects

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	2.514a	5	.503	56.677	.000
Intercept	11.290	1	11.290	1272.679	.000
LEARNING MODEL	.089	1	.089	10.077	.000
KAM	2.208	2	1.104	124.461	.000
LEARNING MODEL * KAM	.001	2	.000	.040	.961
Error	.479	54	.009		
Total	19.583	60			
Corrected Total	2.993	59			

Analysis of Table 20 shows a significance value of 0.000 (sig. < 0.05) for the learning model variable. Based on this result, the null hypothesis (H0) is rejected and the alternative hypothesis (Ha) is accepted. This statistical finding demonstrates that the Problem Based Learning model has a more significant positive influence than the Ordinary Learning Model on enhancing student learning independence. In other words, there is a statistically significant difference between students taught using the problem-based learning approach compared to those taught using conventional teaching methods, with the former showing greater improvement in learning independence.

d) Measuring whether there is no interaction between the learning model and students' initial mathematical abilities on the increase in students' learning independence.

According to the test criteria, the alternative hypothesis (Ha) is accepted if the significance value (sig) exceeds 0.05; otherwise, it is rejected. Analysis of Table 4.21 reveals that the interaction between LEARNING MODEL and Initial Mathematical Ability (KAM) yields a significance value of 0.961 (sig. > 0.05). Consequently, the null hypothesis (H0) is rejected while the alternative hypothesis (Ha) is accepted, indicating that there is no significant interaction between the learning model employed and students' initial mathematical ability regarding the increase in student learning independence. This statistical relationship is visually demonstrated in Figure 2.

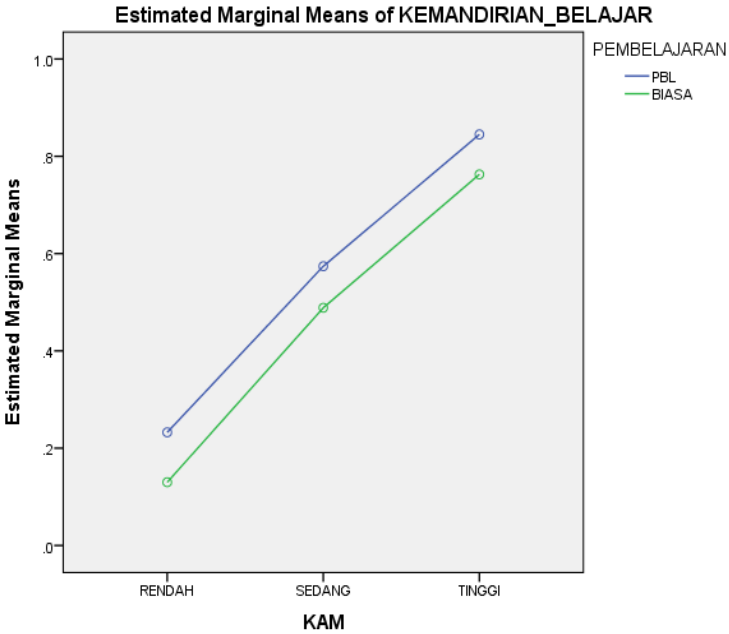

Figure 2. Interaction between the Learning model and Student KAM on Increasing Student Learning Independence

Figure 2 demonstrates that the line representing the experimental class (Problem Based Learning) runs parallel to and does not intersect with the control class line (Regular Learning). This parallel relationship indicates the absence of interaction between the learning model and students' Initial Mathematical Ability (KAM) levels (high, medium, low) in terms of improving students' mathematical critical thinking skills. This finding suggests that there is no combined effect between the learning model and students' KAM on the enhancement of mathematical critical thinking abilities. The explanation lies in the inherent nature of individual student capabilities - students enter the learning environment with varying initial abilities, categorized as high, medium, or low. Students with high initial ability tend to maintain better performance regardless of the teaching methodology employed, and this pattern holds true across all ability levels. In other words, a student's initial mathematical ability remains a consistent predictor of their performance, independent of the learning model used.

CONCLUSION

Based on the research findings, data analysis, and discussion presented in the previous chapter, four main conclusions emerge:

1. Problem Based Learning (PBL) demonstrated superior effectiveness compared to conventional teaching methods in enhancing students' mathematical critical thinking

abilities. Students in the experimental group using PBL achieved a mean score of 71.23, while those in the control group using traditional methods scored 65.53. Moreover, the improvement in critical thinking skills was more substantial in the PBL group (0.527) compared to the control group (0.446). This success can be attributed to PBL's engaging methodology, which involves students directly in information gathering and real-world problem-solving, leading to better long-term retention.

2. No significant interaction was observed between the teaching methodology and students' initial mathematical competency regarding the development of critical thinking skills. This finding reflects the inherent nature of individual student capabilities, where students maintain their relative performance levels (high, medium, or low) regardless of the teaching approach. High-performing students consistently achieve better results irrespective of the instructional method employed.
3. PBL proved more effective than traditional teaching methods in fostering student learning autonomy. The experimental group exposed to PBL recorded a mean independence questionnaire score of 113.9, compared to 103.967 in the control group. The improvement in learning independence was also more pronounced in the PBL group (0.697) versus the control group (0.446). This enhancement can be attributed to PBL's interactive nature, which encourages questioning, suggestion-making, and peer comparison of solutions, thereby stimulating curiosity and independent learning.
4. The research revealed no significant interaction between the teaching approach and students' initial mathematical ability regarding the development of learning independence. This outcome aligns with the observation that students' inherent ability levels (high, medium, or low) tend to correlate with their capacity for independent learning, regardless of the teaching methodology employed.

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