

## PRE-SERVICE MATHEMATICS TEACHERS' ALGEBRAIC THINKING IN SOLVING MATHEMATICS PROBLEMS BASED ON ADVERSITY QUOTIENT

Nur Hanifatur Rahmah<sup>1</sup>, Masduki<sup>2</sup>

<sup>1,2</sup>Universitas Muhammadiyah Surakarta, Surakarta, Indonesia  
e-mail: masduki@ums.ac.id

### Abstract

Algebraic thinking has an important role in solving mathematics problems. In addition, Adversity Quotient (AQ) is one of the factors that can help students succeed in solving math problems. This study aims to investigate the algebraic thinking skills of pre-service mathematics teachers (PMTs) in solving math problems based on adversity quotient (AQ). This study investigates three components of algebraic thinking, namely generalization, functional thinking, and justification. This research used qualitative approach with a case study method. The subjects were 30 PMTs at one of private universities in Surakarta District, Central Java, Indonesia. Data were collected by the algebraic thinking test, ARP (Adversity Response Profile) questionnaires, and interview protocol. The results showed that climber PMTs were able to demonstrate algebraic thinking activities in the components of arithmetic generalization, functional thinking, and generalization and justification. Camper PMTs were only able to demonstrate algebraic thinking activities in the components of arithmetic generalization and also generalization and justification. Meanwhile, quitter PMTs were unable to demonstrate algebraic thinking activities in all components. It can be concluded that the characteristics of AQ are related to the PMTs' algebraic thinking abilities.

**Keywords:** algebraic thinking, adversity quotient, pre-service mathematics teachers

### Abstrak

Berpikir aljabar memiliki peran penting dalam pemecahan masalah matematika. Selain itu, Adversity Quotient (AQ) merupakan salah satu faktor yang dapat membantu keberhasilan siswa dalam menyelesaikan masalah matematika. Penelitian ini bertujuan menyelidiki kemampuan berpikir aljabar mahasiswa calon guru matematika dalam menyelesaikan masalah matematika berdasarkan adversity quotient (AQ). Penelitian ini menyelidiki tiga komponen berpikir aljabar, yaitu generalisasi, fungsional, dan justifikasi. Jenis penelitian ini merupakan penelitian kualitatif dengan pendekatan studi kasus. Subyek dalam penelitian ini sebanyak 30 mahasiswa calon guru matematika pada salah satu universitas swasta di Surakarta, Jawa Tengah, Indonesia. Metode pengumpulan data berupa soal tes berpikir aljabar, angket ARP (Adversity Response Profile), dan wawancara. Hasil penelitian menunjukkan bahwa mahasiswa climber mampu menunjukkan aktivitas berpikir aljabar pada komponen generalisasi aritmatika, berpikir fungsional, serta generalisasi dan justifikasi. Mahasiswa camper hanya mampu menunjukkan aktifitas berpikir aljabar pada komponen generalisasi aritmatika, serta generalisasi dan justifikasi. Sedangkan mahasiswa quitter tidak mampu menunjukkan aktifitas berpikir aljabar pada semua komponen. Dengan demikian, dapat disimpulkan bahwa karakteristik AQ berkaitan dengan kemampuan berpikir aljabar mahasiswa calon guru.

**Kata kunci:** berpikir aljabar, adversity quotient, mahasiswa calon guru matematika

### INTRODUCTION

Relationships in algebra can be expressed through symbols. To express relationships in algebra using symbolic notation requires the ability to think called algebraic thinking. Algebraic thinking is a mental process for reasoning the unknown, and can generalize and formalize the relationship between quantities and the development of variable concepts (van Amerom, 2002). Brown and Herbert (2020) explained that algebraic thinking is the use

of symbols and tools to analyze a different condition by summarizing information from a situation, then presenting that information mathematically in the form of words, graphs, tables, diagrams, and equations. Hereafter, the information is interpreted and applied to mathematical models, such as solving unknown values, investigating proofs, and identifying functional relationships in similar or new situations.

Algebraic thinking is a branch of thinking in mathematics. Kieran (2004) explained that algebraic thinking is the ability to study algebra and representing it in a quantitative situations so that relationships between variables become real. Paridjo (2018) stated that algebraic thinking is grouping information, predicting patterns, and describing rules and changes with different representations, and justifying rules. Based on the aforementioned, it can be concluded that algebraic thinking is the ability to express relationships between objects in mathematics using symbols that represent a certain condition so that the relationship between objects can be clearly known. Understanding the relationships between objects can help make it easier to solve math problems.

Algebraic thinking ability is important for students to master. (Badawi et al., 2016) stated that algebraic thinking needs to be developed by students because with this ability, students will have a focus on relationships and representation in solving mathematical problems. Moreover, Lingga and Sari (2013) also stated that by thinking algebra, students can solve mathematical problems that appear in everyday life.

Kriegler (2007) explained that there are two components in algebraic thinking, namely the mathematical thinking tools and the basic idea of algebra. The mathematical thinking tools include three abilities, namely problem solving, representation, and quantitative reasoning. While the basic ideas of algebra include arithmetic generalizations, algebra as a language, and functions and mathematical modeling. Lew (2004) explained the algebraic thinking indicators consists of six components, namely generalization, abstraction, analytical thinking, dynamic thinking, modeling, and organization. Generalization is the process of making patterns. Abstraction is the process of extracting objects with mathematical relationships based on patterns. Analytical thinking is the process of finding value. Dynamic Thinking is the process of manipulating algebra. Modeling is the representation of a complex situation. Organization i.e. search for value based on modeling. Kieran (2004) classified algebraic thinking includes global generational, transformational, and meta-level activities.

Blanton and Kaput (2011) stated that the components of algebraic thinking include arithmetic generalizations, functional thinking, and generalizations and justifications. In this study, researchers used algebraic thinking indicators developed by Blanton and Kaput (2011) as a framework for investigating the algebraic thinking skills of the PMTs.

The process of solving mathematical problems requires the ability not to give up easily in the face of challenges and difficulties. The ability of a person to manage, administer, and deal with difficulties using their intelligence so that it becomes a challenge to be solved is called the Adversity Quotient (AQ). Students who have a good AQ will be able to survive in solving the complex math problems. Conversely, students who have low AQ will be easily stressed and give up, and also despair in solving complicated math problems. Therefore, AQ has an important role in the ability of students to deal with problems, especially solving mathematical problems. Stoltz (1997) grouped the AQ into three categories, namely Climber, Camper, and Quitter. The Climber category is a person who always strives to reach the pinnacle of success and ready to face existing obstacles. The Camper category is a person who is easily satisfied with what has been achieved so that he cannot reach the peak of success. While the Quitters category is a person who chooses to avoid and reject existing opportunities, easily discouraged, and easily gives up so that he is not eager to reach the success. These shows that students' ability to overcome difficulties in solving math problems also varies (Masfingatin & Murtafi'ah, 2016).

Research on algebraic thinking processes reviewed from AQ has been studied. Sanit et al., (2019) found that Climber students are able to perform algebraic thinking processes at generalization, transformations and meta global levels activities. Then, Camper's students were only able to perform algebraic thinking processes on generalization activities. Quitter's students were unable to perform algebraic thinking processes on all algebraic thinking activities. Research by Amalliyah et al. (2022) found that algebraic thinking skills of Quitter's students demonstrated by writing the known information from problems, determining variables, and making mathematical models. Camper students can meet indicators of algebraic thinking skills, namely determining variables, compiling mathematical models, using mathematical models to solve problems, and determining variable values as solutions to problems. Then, Climber students can meet all indicators of algebraic thinking ability.

Research on algebraic thinking skills based on the AQ category in pre-service mathematics teachers (PMTs) is still relatively limited. As prospective teachers, PMTs are require the intelligence not to give up easily and be able to face various challenges, especially in solving complex mathematical problems. The teacher's intelligence in managing difficulties can affect on students mental. The teachers who give up easily in solving complex math problems, of course students will also give up easily in solving math problems, and vice versa. Therefore, this study aims to reveal the algebraic thinking ability of PMTs in solving math problems associated with AQ.

## METHODS

This research approach is qualitative with a case study design. The subjects of this study were 30 third-year PMTs at one of the private universities in Surakarta, Central Java, Indonesia. This study used three instruments, namely ARP (Adversity Response Profile) questionnaires, algebraic thinking test questions, and interview protocols. The ARP questionnaire developed by Stoltz (1997) used to categorize AQ PMTs. ARP contains 30 questions grouped in the dimensions of control (C), origin (OR) and ownership (Ow), reach (R), and endurance (E), or in the form of the acronym CO2RE. Based on the results of the ARP questionnaire, PMTs are grouped into three AQ categories, namely high AQ (Climber), medium AQ (Camper), and low AQ (Quitter). Afterwards, PMTs' AQ were categorized as presented in Table 1.

**Table 1. Summary of PMTs' AQ**

Score	Category	Multiple Students
≤95	Quitter	1
96-134	Camper	23
≥135	Climber	6

Subsequently, researchers compiled eight algebraic thinking problems adopted from several literature as a test instrument (Blanton & Kaput, 2011; Hodnik Čadež & Manfreda Kolar, 2015; OECD, 2013). The instrument consists of two arithmetic generalization problems, three functional thinking problems, and three generalization and justification problems. Before being used, the test instrument was validated by two mathematics education experts and piloted on 12 students in addition to the subjects who would

participate in the research. After improvement, the instrument is ready to be used for research data collection. In this paper, the researchers presents an analysis of three algebraic thinking problems that include the components of arithmetic generalization (Q1), functional thinking (Q2), and generalization and justification (Q3) as presented in Table 3.

**Table 3. Algebraic Thinking Test Questions**

No	Question
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Q1

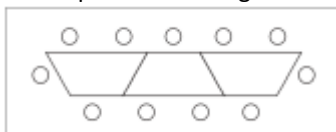


Cars made of grapefruit peel are one of the traditional toys of Indonesian children. A toy craftsman wanted to make cars using orange peels to share with children around his house. The materials necessary to create the cars are presented in the table below:

Material	Stick	Orange peel for car body	Tire-shaped orange peel
Materials are needed to make a car	3	2	4
Materials available	27	19	30

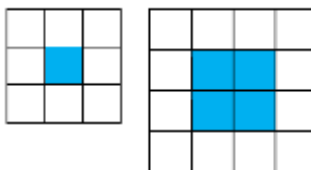
Make sure of the number of cars that can be made from the available materials!

Q2 In a meeting room, there are several trapezoid-shaped tables. The seating position of the person for three sequentially arranged tables is presented in Figure.



- Determine the number of people who can be seated if there are 20 tables arranged as pictured! Explain of your solution steps!
- If there are 40 people present, determine the number of tables needed! Explain of your solution steps!
- Determine a formula that expresses the relationship between the number of tables and people who can sit!

Q3 A construction worker will create a pattern of white and blue tiles with the rules as presented in Figure below.



Specify:

- The number of blue tiles are needed to arrange the tile pattern as shown in Figure if 52 white tiles are available!
- The number of white tiles are needed to compose the tile pattern as shown in Figure if 64 blue tiles are available!
- Define a formula that expresses the relationship between the number of blue tiles and white tiles!

To analyze the PMTs' answers sheet, the researchers used the assessment rubrics as presented in Table 4.

**Table 4. Algebraic Thinking Test Results Assessment Rubric**

Value	Information
0	Did not answer or incorrect answer
1	The solution step is partially correct but the answer is incorrect
2	The solution step is correct but the answer is incorrect
3	The solution steps and answers are correct

Researchers selected five subjects, coded by S1, S2, S3, S4, and S5 considering the scores of algebraic thinking test instrument and ARP questionnaire scores. The profiles of the five subjects of the study are presented in table 5.

**Table 5. Research Subject Profile**

Code	AQ Categories	ARP Score	Algebraic Thinking Test Score
S1	Climber	160	46
S2	Climber	140	43
S3	Camper	111	44
S4	Camper	110	42
S5	Quitter	70	15

For completing the analysis, the subjects were interviewed to further reveal the PMTs' process and strategies in solving algebraic thinking test problems. The results of the interview and the document analysis were triangulated as data validation related to PMTs' algebraic thinking abilities and processes.

## RESULTS AND DISCUSSION

This study analyzed the algebraic thinking skills of students with the categories of Climber, Camper, and Quitter. Analysis of students' algebraic thinking skills uses three components of algebraic thinking, namely arithmetic generalization, functional thinking, and generalization and justification.

### Arithmetic Generalizations

The Q1 problem is used to reveal students' algebraic thinking skills in solving problems related to arithmetic generalizations, namely the ability to arrange relationships between quantities in problems. There are two subjects selected in the Climber category, S1 and S2. Based on the analysis of the answer sheet document and the results of the interview, both subjects used relatively similar solution steps. Figure 1 shows an example of S1's answer to problem Q1.

Bahan	Udi	Kulit jeruk utk badan mobil	Kulit jeruk bentuk ban
Yang diperlukan	3	2	4
Yang tersedia	27	19	30

Tentukan banyak mobil - mobilan yg dapat dibuat dr bahan yg tersedia!

Jawab:

Misalkan :  $x$  = banyaknya Udi  
 $y$  = banyak kulit jeruk untuk badan mobil  
 $z$  = banyak kulit jeruk bentuk ban

$$3x = 27 \quad 2y = 19 \quad 4z = 30$$

$$x = 9 \quad y = 9,5 \quad z = 7,5$$

Jadi, banyak mobil - mobilan yg dapat dibuat dari bahan yang tersedia yaitu dapat membuat 7 mobil dengan sisa bahan yang tersedia antara lain Udi = 2, badan mobil = 2,5, dan ban = 0,5

Figure 1. S1 Answer of Q1

Figure 1 shows that S1 solve the Q1 problem by presenting the known information on the problem in the form of a table. S1 compiles a table in the form of stick, orange peel for the car body, orange peel for the required and available tires. Then, S1 presents the data in the table in the form of variables. S1 uses variable  $x$  To express the number of sticks,  $y$  to express the number of orange peels of the car body, and  $z$  to express the amount of orange peel on a car tire. Afterwards, S1 presents the relationship between the required material and the available material in the form of a linear equation. In this case, S1 presents the relationship of the stick material in the form of  $3x = 27$ , The relationship of orange peel material to the body in the form of  $2y = 19$ , and the relationship of orange peel material to car tires in the form of  $4z = 30$ . Using these equations, S1 determine the number of sticks, car bodies from orange peels, and car tires from orange peels i.e.  $x = 9$ ,  $y = 9,5$ , dan  $4z = 7,5$ . Thus, S1 concluded that the number of cars can be made from the available materials as many as 7 cars with the remaining materials 2 sticks, 2.5 orange peels of the car body, and 0.5 orange peels of car tires. Based on the results of the analysis of answer documents, S1 can compile relationships between quantities in the problem. In other words, S1 shows the ability to solve problems related to arithmetic generalizations.

Furthermore, the researcher conducted an interview with S1 to reveal the subject's understanding of the problem solving process. The following is presented an excerpt from the researcher's interview with S1.

P : How do you answer question number 1?

S1 : For number 1, I use the example

P : What kind of example?

- S1 : Use variables. For example, a lot of sticks =  $x$ , the abundance of orange peels for the car body =  $y$ , and the abundance of orange peels for car tires =  $z$
- P : After using variable, what is the next step?
- S1 : Created equations, the first  $3x = 27$  for the sticks, the second  $2y = 19$  for the body of the car, and the third  $4z = 30$  for car tires
- P : Is it done until that's your answer?
- S1 : No, after creating the equation, I look for the value  $x, y$  and  $z$ . By moving the side and division,  $3x = 27$  so  $x = 27:3$ , the result is 9 sticks,  $2y = 19$  so  $y = 19:2$  the result is 9.5 car bodies, and  $4z = 30$  so  $z = 30:4$  The result is 7.5 car tires.
- P : Okay, after that?
- S1 : Because to make one car needs 3 sticks, 2 orange peels for the car body, 4 orange peels for car tires, and it has been calculated for the stick as many as 9, orange peel for the car body there is 9.5, and orange peel for car tires there is 7.5. So, the number of cars that can be made based on the available materials are 7 cars, the remaining 2 sticks, 2.5 orange peels for car bodies, and 0.5 orange peels for car tires.

The interview results showed that S1 was able to represent information on the problem in the form of variables, namely  $x$  for stick,  $y$  for orange peel of the car body, and  $z$  for orange peel of car tires. S1 is also able to compile equations related to the relationship between available materials and the required materials and determine the solution of the equation. The interview results showed that S1's understanding of the Q1 problem solving process was consistent with the results of the analysis of the answer sheet document. Thus, it can be concluded that the subject Climber is able to solve problems related to arithmetic generalizations, namely compiling relationships between quantities.

There are two subjects selected in the Camper category, namely S3 and S4. The results of document analysis, answer sheets, and interviews showed that both subjects also used relatively the same Q1 question solving steps. Figure 2 shows an example of S3's answer to problem Q1.



① Tentukan banyak mobil-mobil yg dapat dibuat

Jika diket :

⇒ yg dibutuhkan membuat sebuah mobil :

Lidi = 3  
kulit jeruk 4/ badan mobil 2  
————— 11 ————— ban mobil 4

⇒ bahan tersedia

Lidi = 27  
kulit jeruk 4/ badan mobil 19  
————— = ————— ban mobil 30

Penyelesaian

Lidi =  $\frac{27}{3} = 9$

kulit jeruk 4/ badan mobil =  $\frac{19}{2} = 9,5 \approx 9$

kulit jeruk 4/ ban mobil =  $\frac{30}{4} = 7,5 \approx 7$

Jadi mobil-mobilan yg dapat dibuat dr bahan diatas adalah 7 buah mobil dilihat dari kesediaan ban mobil

Figure 2. S3 Answer of Q1

Figure 2 shows that, in solving problem Q1, S3 does not write the solving steps using variables, but explains the solving steps using sentences based on the information in the problem. S3 performs division operations directly using the numbers available in the problem. Figure 2 also show that the S3 can determine the number of cars that can be made based on the availability of materials, 7 cars. The following interview excerpts also show that S3 divides directly using available numbers.

P : After you look up what you know and ask, how do you solve question number 1?

S3 : I use the method of dividing one by one, so the stick is 27 divided by 3 because what is needed 3 becomes 9, then I can make 9 cars. For the orange peel it was 19 divided by 2, then the car tire skin was 30 divided by 4 the result was 7.5 rounded to 7. Then why here I conclude that a car can be made as many as 7, because I see it from the tires of the car that are made into a reference to be a car.

Based on the results of document analysis and interviews, S3 is able to explore arithmetic operations and relations to solve mathematical problems related to arithmetic generalization. However, S3 does not use variables to represent the relationship between

quantities in the problem. In other words, Camper's subject can solve problems related to arithmetic generalizations.

There is only one subject that represents PMTs with the Quitter category, namely S5. The results of document analysis and interviews showed that S5 was unable to solve problems related to arithmetic generalizations. The S5 simply writes down the final results the number of cars that can be made based on the availability of materials without using solution steps. In the interview, S5 was also unable to explain how answers could be obtained.

The results of the analysis of answer documents and interviews showed that climber and camper subjects were able to use arithmetic operations and relations to solve mathematical problems related to arithmetic generalizations. However, the strategies used by climber and camper subjects differed. The climber subject represents the information on the problem in the form of variables, then compiles the relationships between variables in the form of linear equations. While the camper subject solves the problem by directly performing the operation of the numbers available in the problem.

### **Functional Thinking**

The Q2 problem is used to reveal functional algebraic thinking skills, which is the process of finding functional relations and symbolizing the quantities. In climber subjects, S1 and S2, the results of the analysis of answer sheet documents and interviews showed that both subjects used relatively similar on Q2 problem solving steps. An example of the answer of subject S2 in solving Q2 is presented in Figure 3.

1. a) Jmlh orang yang dapat duduk jika tersedia 20 meja  
 Ditetahui pada gambar bahwa setiap meja width ada 3 kursi  
 dan 1 kursi sebelah kiri, dan 1 kursi sebelah kanan  
 Jadi ~~banyak~~ jumlah kursi = jumlah meja  $\times$  3 kursi + 2  
 Maka jmlh orang yang dapat duduk jika tersedia 20 meja adalah  

$$\begin{aligned} 20 \times 3 &= (20) \times 3 + 2 \\ &= 60 + 2 \\ &= 62. \end{aligned}$$
- b) Diket : 40 orang hadir  
 Ditanya : banyak meja yang diperlukan  
 Jawab :  
 Jika 1 meja = 3 kursi = 3 orang  
 Maka dibutuhkan 13 meja  $\Rightarrow$   $13 \times 3$  orang = 39 orang.  
 Karena setiap sisi kanan terdapat 1 meja samping kiri dan  
 1 meja samping kanan.  
 Maka  $39 + 2 = 41$  kursi.  
 $\hookrightarrow$  sudah memenuhi jmlh orang yg hadir.  
 Jadi dibutuhkan 13 meja.
- c) Rumus  
 $\hookrightarrow 3x + 2$   
 ket  $\Rightarrow$  3 : jmlh kursi yg harus ada pd tiap meja  
 $x$  : jmlh meja  
 $2$  : jmlh kursi samping kiri dan kanan pd tiap pojok.

Figure 3. S2 Answer of Q2

Figure 3 shows that to solve the Q2.a problem, S2 determines that on each table there are 3 chairs and on the right and left sides of the row of tables there is 1 seat each. Furthermore, to determine the number of people who can sit for 20 tables available, S2 uses the formula for the number of tables multiplied by 3, namely  $20 \times 3 = 60$ . Then the multiplication result, which is 60, plus 2, namely the seats on the right and left on the row of tables obtained 62. Thus, S2 concluded that there are 62 people who can sit for the 20 tables available.

To solve the Q2.b problem, S2 determines that each table has 3 people who can sit. Thus, the number of tables needed for 40 attending people are 13 tables. S2 explained that for each table there are 3 people who can sit in chairs. So, the the number of people who can sit, can be determined by multiplying the number of tables, i.e. 13, with the people who can sit in each table, i.e. 3, obtained 39 people. Furthermore, because on each right and left side of the row of table there is 1 seat each, so the number of people who can sit on the row

tables is 41 people. Thus, S2 concluded that the number of tables to accommodate 40 people is 13 tables.

For Q2.c problem, S2 uses the equation  $3x + 2$  to express the relationship between the number of tables and the attending people. The coefficient 3 represents the number of chairs on each table, the variable  $x$  represents the number of tables, and the constant 2 represents the the number of chairs on the right and left side of the table row. Thus, S2 able is to determine the relationship between the attending people and the number of tables required. In other words, S2 is able to determine a functional relationship that is a relation between two generalized quantities in the form of functions.

Subsequently, researchers conducted an interview with S2 regarding the process of solving Q2. Excerpts of the researcher's interview with S2 are presented as follows.

P : How do you answer question number 2?

S2 : I suppose  $x$  is the number of tables, coefficient 3 is the number of seats that each table should have, and coefficient 2 is the number of seats on the right and left sides and I use the formula  $3x + 2$  for number 2.

P : How?

S2 : point A because the known 20 tables are so  $3 \times 20 + 2$  The result was 62, for point B because there were 40 people present and asked about the number of tables. So many of the number of tables needed are as many as 13, which I use  $3x + 2 = 41$ . So that  $x = 13$

Based on the results of the analysis of answer documents and interviews, S2 is able to use functional relationships, namely the relation between two quantities in the form of functions to solve problems related to functional thinking. In other words, the Climber subject is able to demonstrate the ability to solve problems related to functional thinking.

For Camper students, S3 and S4, the results of document analysis and interviews showed that the subjects could not solve Q2 questions correctly. Figure 4 is an example of the results of the S2 answer to question Q2.

a) ~~30~~ 3 meja saja ada beangbinan ada 11 porsi duduk orang  
 mata, untuk 20 meja yaitu  $\frac{20}{3} = 6,67$  |  
 kemudian, 6,67 dikalikan 11 = 73,37 dibulatkan 74  
 Jadi, kemungkinan jumlahnya ada 74 orang yang  
 dapat duduk.

b) Berapakah dipin (a) 3 meja ada 11 orang  
 mata, untuk 40 orangnya yaitu  $\frac{40}{11} = 3,63$  |  
 kemudian, 3,63 dikalikan 3 meja adalah 10,89 dibulatkan 11  
 Jadi, banyak meja yang diperlukan adalah 11 meja

Figure 4. S4 Answer of Q2

Figure 4 shows that S4 was unable to solve the Q2 problem correctly. S4 solves the Q2.a problem by determining the approximate number of people who can sit for 3 available tables, i.e. 11 people. Then S4 divides the available tables by 3 to obtain 6.67. Afterwards, S4 multiplies 6.67 by the estimate of a person who can sit at 3 tables, i.e. 11 to obtain 73.37, rounded up to 74. Because the solution to the Q2.a question is incorrect, the next solution of the problems Q2.b and Q2.c, is also incorrect. The solution step showed that S4 subject has not understood the functional relationship between two quantities, in this case the relationship between the number of tables and people who can sit. The interview results also showed that S4 could not explain the appropriate solution steps. Thus, it can be concluded that the Camper subject unable to demonstrate functional thinking abilities, that is, has not understood the functional relationship between two quantities.

For Quitter subject, S5 is unable to answer Q2 problem. The interview results also showed that S5 could not understand the meaning of the Q2 problem. In other words, the S5 is unable of demonstrating functional thinking skills. Thus, based on the results of the analysis of answer documents and interviews, only the Climber subjects were able to demonstrate functional thinking abilities by connecting between two quantities and then generalized in the form of functions.

### Generalizations and Justifications

Q3 problem is used to reveal students' algebraic thinking skills in the generalization and justification component, namely students' ability to find a shape or pattern. In Climber and Camper subjects, based on the results of document analysis of answer sheets and

interviews, all four subjects used relatively the similar solution steps. Figure 5 is an example of the answer from S1 to solve Q3 problem.

a. Banyak ubin biru yg diperlukan jika tersedia 52 ubin warna putih

1 = 9 putih 1 biru  
2 = 12 putih 4 biru  
3 = 16 putih 9 biru

$a = 0$   
 $b = 4$   
 $U_n = a + (n-1)b$  mencari putih  
 $U_n = n^2$  mencari biru  
 $52 = 0 + (n-1)4$   
 $49 = 4n - 4$   
 $48 = 4n$   
 $12 = n$

Jadi, banyak ubin biru yg diperlukan  $12^2 = 144$

b. Ubin putih jika tersedia 64 ubin biru

$64 = 8^2 \rightarrow$  suku ke 8

$U_n = a + (n-1)b$   
 $U_8 = 0 + (8-1)4$   
 $= 0 + 7 \cdot 4$   
 $= 36$

Jadi, ubin putih yg diperlukan yaitu 36

c. Tentukan rumus

$U_n = a + (n-1)b$  ubin putih  
 $U_n = n^2$  ubin biru

Figure 5. S1 Answer of Q3

Figure 8 shows the S1 solution steps of Q3.a problem. S1 draws the pattern available on the problem to determine the number of blue tiles and white tiles. Then, S1 identifies the number of white and blue tiles on each pattern. Based on the identification results, S1 concludes that the value  $a = 8$  and  $b = 4$ . Next, S1 uses the arithmetic sequence formula  $U_n = a + (n - 1)b$  to specify the number of blue tiles for 52 available white tiles. In the same way, S1 solves the Q3.a and Q3.b problems by arithmetic sequence formula. Thus, based on the analysis of the document, it can be concluded that S1 can demonstrate the ability to determine number patterns based on the available data. The interview results also showed that S1 was able to explain the use of arithmetic sequence formulas to solve Q3 problems.

P : How do you solve problem 3?

S1 : If those who are looking for white tiles use ordinary arithmetic formulas, then if those who are looking for blue tiles use formulas  $Un = n^2$ , follow the pattern. Point a first looks for the white tile, so I use the formula  $Un = a + (n - 1)b$  with  $a = 8$ ,  $b = 4$ , and  $Un = 52$ , so that the result  $n = 12$  then squared to 144 for the blue tile. While point b is known to be 64 for blue tiles, so  $n^2 = 64$  Result  $n = 8$ . Next to find the white tile using the formula  $Un = a + (n - 1)b$  by way of substitution  $n = 8$ ,  $a = 8$ , and  $b = 4$  result 36

The results of the analysis of answer documents and interview showed that S1 is able to solve problems related to generalization and justification appropriately. In other words, the subjects of Climber and Camper are able to demonstrate the ability to solve problems related to generalization and justification appropriately. In this case, the subject can determine the pattern of numbers using arithmetic sequences.

For the Quitter subject, the results of the analysis of answer documents and interviews showed that the subject was unable to determine the pattern from the data in the Q3 problem. In other words, the Quitter subjects was unable in performing algebraic thinking activities related to generalization and justification.

### Discussion

The results of the analysis showed that the Climber subjects were able to show algebraic thinking activities in the three components investigated, namely arithmetic generalizations, functional thinking, and generalizations and justifications. This is in line with the study by Amaliyah (2022) which showed that climber students can meet all indicators of algebraic thinking ability. The Climber subjects has a strong pretension and motivation to solve and analyze the problem. The Climber subjects also do not give up easily and always try to find answers to the questions. This is reinforced by Pebriana et al. (2019) who explained that people with the Climber category are people who always try to reach the pinnacle of success and ready to face the obstacles given so as to get the best results.

Subsequently, the Camper subjects were only able to show algebraic thinking activity on the components of arithmetic generalizations as well as generalizations and justifications. This is in line with research by Sanit et al. (2019) that Camper's students in algebraic thinking only use generalizations. Camper subjects have the ability to struggle in

the face of adversity but have not been able to optimize their potential due to the character of Camper subjects who are easy to give up and despair. The statement is reinforced by Marpaung (2007) which stated that Camper students have the characteristics of being easily discouraged by what has been achieved and have low motivation in learning.

The finding also found that the Quitter subject was unable to show algebraic thinking activity on all components of algebraic thinking. Quitter subjects tend to be easily discouraged and do not have the motivation to try to solve a given problem. This is in line with the results of the study by Sukmaningrum and Kurniasari (2022) which concluded that Quitter's students are not capable of doing all indicators of algebraic thinking. The characteristics of Quitter's subject are also in line with the statement from Stoltz (1997) which stated that people with the Quitter type tend to be easily discouraged, give up easily and are not eager to reach the pinnacle of success. People with the Quitter type need guidance in developing their potential to achieve success. The statement is also supported by Listiawati and Sebayang (2019) which states that appropriate action for students who are lacking, i.e. with directed guidance to improve their adversity intelligence.

Suryadi and Santoso (2017) revealed that adversity quotient is one of the factors that influence student learning outcomes. Thus, increasing student adversity quotient needs to be considered by the teachers to improve student learning outcomes. In addition, the learning strategies used by the teacher have also an impact on improving student AQ. Therefore, teachers must be creative and innovative to develop learning strategies that can increase students' AQ (Amir et al., 2021). The findings of this study show that the characteristics of adversity quotient are influential in character building and the thinking ability. This is also supported by the results of the study by Nahrowi et al. (2020) that adversity quotient can be used to identify students' ability to solve problems. Adversity quotient needs to be considered by teachers, especially to solve math problems. The adversity quotient has a positive influence on the student's achievement (Rahayu & Alyani, 2020). In addition, the adversity quotient is also considered better for knowing one's success (Juwita et al., 2020).



## CONCLUSION

Adversity quotient indicates the fighting power to face challenges and difficulties, especially in solving mathematical problems. Climber subjects show unyielding struggle power in solving problems related to algebraic thinking. This is shown by the ability to solve problems related to arithmetic generalizations, functional thinking, and generalization and justification appropriately. Furthermore, subjects with the Camper type showed the struggling power that was easily satisfied with what had been achieved. This is shown by the ability to solve problems correctly in two components of algebraic thinking, namely arithmetic generalization and generalization and justification. While subjects with quitter type, tend to give up easily and are less motivated in solving problems. Quitter's subjects were unable to demonstrate problem-solving skills in all three components of algebraic thinking.

The findings of this study show that the characteristics of a person's struggling power have an impact on the ability to think in solving mathematical problems. Therefore, PMTs must have high struggling power in order to guide students in solving complex mathematical problems. In addition, research to identify factors that affect the type of AQ is also important to do.

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