EXPLORATION OF STUDENTS' ALGEBRAIC THINKING SKILLS IN SOLVING TIMSS PROBLEMS IN TERMS OF REFLECTIVE-IMPULSIVE COGNITIVE STYLE

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Abstract
The algebraic thinking is very important for students to be able to abstract and generalize a mathematical problem. The purpose of this research is to reveal students’ algebraic thinking processes in solving math problems based on reflective-impulsive cognitive styles. The method used is a qualitative with a case study approach. The research subjects consisted of 55 students who were then selected six students based on the reflective-impulsive cognitive style. The research instrument used consisted of an algebraic thinking test that adopted from TIMSS (Trends in International Mathematics and Science Study), MFFT (Matching Familiar Figure Test) and an interview guide. In this paper, researchers are focus on the exploration of students’ algebraic thinking ability in solving problems on generalization and abstraction components. The finding showed that the reflective subjects and impulsive subjects are able to solve generalization and abstraction problems. The reflective and impulsive subjects are able to use patterns to solve problems related to generalizations. In addition, reflective and impulsive subjects are able to use symbols as an abstraction of the relationship between concepts and mathematical properties. However, the reflective subjects used the solution problem steps more systematic than impulsive subjects. The implication of the finding is that the individual differences such as cognitive style should be facilitated by the teachers so that students are more successful in learning mathematics.

Keywords: algebraic thinking, cognitive style, reflektif-impulsif, generalization, abstaction

INTRODUCTION
Algebraic thinking is a thinking process that involves algebraic symbols as tools such as analyzing quantitative relationships, generalization, problem solving, modeling, predicting and drawing conclusions (Kieran, 2004). Van de Walle (2004) stated that algebraic thinking
includes generalizing activities from calculations in numbers, using useful symbols and exploring functions and patterns. Algebraic thinking is a skill that arises from the process of thinking activities that include generalization, abstraction, dynamic reasoning, analytical reasoning, and modeling (Lew, 2004). Ntsohi (2013) mentioned that algebraic thinking is the activity of using mathematical symbols and tools to analyze different conditions, represent mathematical information in the form of words, diagrams, tables, graphs and equations, and prove and determine relationships between functions.

Kieran (2004) explained that indicators of algebraic thinking ability include generalizations involving the formation of expressions and equations in algebraic objects, transformations related to changes in expressions and rule-based equations, and the metaglobal level by involving algebra as a tool to solve a problem. According to Lew (2004), student success in algebraic thinking is determined by six components, namely generalization, abstraction, analytical thinking, dynamic thinking, modeling and organization. On the other hand, Blanton dan Kaput (2005) explained that the components of algebraic thinking in direct and planned form in classroom learning include arithmetic generalizations, functional relationships, number properties, operations, and algebraic treatment of numbers.

The ability to think algebraically encourages a person to be able to abstract and generalize a problem (Foster, 2007). The ability to think algebraically encourages a person to be able to abstract and generalize a problem (Booker dan Windsor, 2010). Through algebraic thinking students can perform activities of analyzing, presenting and generalizing symbols, patterns and numbers presented in the form of tables, graphs and mathematical expressions (Laisouw, 2012). Algebraic thinking is inseparable from students' ability to solve a problem such as a story problem (Phonapichat et al., 2014). Problem-solving ability is the ability of students to understand, choose strategies, and carry out solving strategies for given problems (Sari et al., 2020). By solving problems, students learn to apply math skills in new ways, develop a deep understanding of mathematical ideas, and learn how to become mathematicians (Badger et al., 2012).

Research related to algebraic thinking of junior high school students has been conducted by several researchers. Research conducted by Agoestanto et al., (2019) regarding algebraic thinking of junior high school students on the components of
generalization, transformation and meta-global concluded that there is a positive influence between the components of generalization, transformation and meta-global in students' mathematics learning. Then, Arina and Lukito, (2019) in his research related to the ability to think algebraically of junior high school students in solving problems in terms of cognitive style, it was concluded that systematic subjects were more coherent in solving problems. While the intuitive subject solves the problem based on its level of difficulty by doing trial-error. Furthermore, research by Harti and Agoestanto (2019) showed that the level of critical thinking and algebraic thinking skills met the minimum criteria. Students with high critical thinking skills have high generalization, transformation and meta-global skills. While students with moderate critical thinking skills have moderate algebraic thinking skills. Furthermore, students with low critical thinking skills tend to have low algebraic thinking skills. Zaelani et al., (2020) in his research on algebraic thinking of junior high school students in terms of SOLO taxonomic concluded that students are able to use variables as generalizations to numbers. Then Safitri and Masduki (2023) in his research related to algebraic thinking in junior high school students in terms of intuitive cognitive style concluded that students of intuitive cognitive style are able to meet the components of generalization, analytical thinking and dynamic thinking. The results of the previous study showed that students' algebraic thinking skills are influenced by the students' cognitive styles.

There is a classification of cognitive styles according to Kozhevnikov (2007) that are based on psychological cognitive style, conceptual cognitive style, and cognitive style of way of thinking. Psychological cognitive styles include field-dependent and field-independent cognitive styles. Then, conceptual cognitive styles include reflective and impulsive cognitive styles. Furthermore, cognitive styles of ways of thinking include intuitive-inductive and logical-deductive cognitive styles. Rozencwajg and Corroyer (2005) explains that the reflexive-impulsive cognitive style is defined as a type of cognitive system that combines decision-making time and student performance in problem-solving situations that contain a high degree of uncertainty.

The results of previous studies related to the investigation of students' algebraic thinking skills associated with reflective-impulsive cognitive styles are still relatively limited.
Therefore, this study aims to investigate students' algebraic thinking skills in terms of reflective-impulsive cognitive styles. This paper focuses on algebraic thinking skills, components of generalization and abstraction. This research provides benefits in mathematics learning that each student has a different cognitive style. Therefore, teachers must be able to understand the characteristics of each student in order to learn more optimally.

METHODS

This research is qualitative research with a case study approach. The subjects of the study consisted of 55 grade VIII students of one of the private junior high schools in Surakarta. The research instruments used include algebraic thinking test questions, MFFT cognitive style instruments and interview guidelines. Researchers compiled an algebraic thinking test question instrument by adopting the 2011 TIMSS questions for grade VIII (Provasnik, 2013). Researchers compiled ten questions consisting of 2 questions each in each component of algebraic thinking according to Lew (2004). Before use, the questions were first validated by 2 mathematicians and 5 mathematics learning experts. Based on the validation results, the researcher set eight questions by eliminating 1 question in the components of generalization and dynamic thinking. Furthermore, researchers conducted a test question on 20 students in addition to the subject to be researched. Based on the results of the trial, researchers made improvements to the problems used for data collection. Then, to classify the students' cognitive styles, researchers used MFFT instruments from Warli, (2010) which has been tested for validity and reliability. Furthermore, to uncover students' algebraic thought processes in depth on each cognitive style, researchers used interview guidelines. Before use, interview guidelines are first validated by expert validators.

In this study, three questions will be analyzed consisting of 1 generalization question and 2 abstraction questions as presented in Table 1. The generalization problem aims to investigate students' ability to determine patterns based on known number patterns. Then, the abstraction problem aims to investigate students' ability to determine the rectangular area and the area of the shaded area.

<table>
<thead>
<tr>
<th>No</th>
<th>Question Type</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Generalization</td>
<td>Several pieces of tiles are arranged into a square shape as follows:</td>
</tr>
</tbody>
</table>
The 3 x 3 square shape consists of 8 Grey tiles and 1 black tile. Then, the 4 x 4 square shape consists of 12 Gray tiles and 4 black tiles.

The table below shows the number of tiles arranged into square shapes of various sizes. Complete the table below to find out the number of tile pieces that make up the square.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Number of Black Tiles</th>
<th>Number of Gray Tiles</th>
<th>Total Tiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 x 3</td>
<td>1</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>4 x 4</td>
<td>4</td>
<td>12</td>
<td>16</td>
</tr>
<tr>
<td>5 x 5</td>
<td>9</td>
<td>16</td>
<td>25</td>
</tr>
<tr>
<td>6 x 6</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 x 7</td>
<td>25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2 Abstraction: Take a look at the picture below!

Determine the area of the rectangle above?

3 Abstraction: Take a look at the rectangle image below!

If the width of the unshaded area is 1m, then determine the equation showing the area of the shaded area (m$^2$)!

Furthermore, based on the results of the algebraic thinking test and MFFT questionnaire to 55 students, recapitulation data were obtained as presented in Table 2.

<table>
<thead>
<tr>
<th>No</th>
<th>Cognitive Styles</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Reflective</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>Impulsive</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>Fast Accurate</td>
<td>39</td>
</tr>
<tr>
<td>4</td>
<td>Slow Inaccurate</td>
<td>1</td>
</tr>
</tbody>
</table>
Based on the data in Table 2, researchers selected 6 subjects consisting of 3 subjects in the reflective cognitive style category and 3 subjects in the impulsive cognitive style category. Subject selection is based on the speed at which the questionnaire is completed and the frequency of errors made by the subject. To facilitate data analysis, reflective subjects were assigned S1, S2, and S3 codes, and impulsive subjects were assigned S4, S5, and S6 codes.

Data analysis is carried out by first conducting document analysis, namely analysis of students' answers in solving algebraic thinking test questions. The focus of the analysis is on the steps of solving the problem and the strategy used to solve the problem. In document analysis, researchers use assessment rubrics as presented in Table 3.

![Table 1. Assessment Rubric](image)

<table>
<thead>
<tr>
<th>No</th>
<th>Judging Criteria</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Completion steps and correct answers.</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Right resolution steps and wrong answers.</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>Partial correct resolution steps and incorrect answers.</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Incorrect resolution steps or unable to answer the question.</td>
<td>0</td>
</tr>
</tbody>
</table>

Furthermore, researchers conducted interviews to dig deeper into students' steps and solving strategies in solving algebraic thinking problems. The interview is also meant to validate the student's answers in the document i.e. the student's answer sheet.

**RESULTS AND DISCUSSION**

In this section, the results of document analysis and interviews are presented 6 subjects of reflective and impulsive cognitive styles in solving generalization and abstraction problems.

**Generalization.**

Question number 1 is used to explore students' ability to generalize, which is a process to find a pattern of pictures or numbers based on known patterns. Based on the test results, subjects S1, S2, S3, S5 and S6 can solve the questions using the right solving steps and get the right answers as well. Only S4 subjects are unable to solve the problem using the right solving steps so that the answers obtained are not correct. The answer of subject S1 in solving question number 1 is presented in Figure 1.
Based on Figure 1, S1 solves the problem by first determining the difference in the number of black tiles on each square shape. S1 can determine the difference in the number of black tiles of 4 x 4 and 3 x 3 square shapes is 3, the difference in 5 x 5 and 4 x 4 square shapes is 5, the difference in 6 x 6 and 5 x 5 square shapes is 7, and the difference in 7 x 7 and 6 x 6 square shapes is 9. Then, S1 can also determine the difference in gray tiles on each known square shape, which is 4. Thus, S1 can determine the number of gray tiles on a 6 x 6 square shape is 20 and a 7 x 7 square shape is 24. Furthermore, S1 can determine the total tiles on each square shape by summing many black and gray tiles on each square shape. S1 says the total number of tiles of a 6 x 6 square shape is 36 and a 7 x 7 square shape is 49.

Next, the researcher conducted an interview with S1 regarding the answer to question number 1. Excerpts of the interview with S1 are presented as follows.

Researchers: How do you solve question 1?

S1: Note the shape of the 3 x 3 tile with a total of 9 tiles, then if the 4 x 4 shape has 16 total tiles, 5 x 5 total tiles are 25, then if the shape of the 6 x 6 tiles the total tiles are 36 and 7 x 7 is 49. As for the number of gray tiles, note the difference from 8 to 12 is 4 so that 8,12,16,20 and 24 are obtained.

The interview results show that S1 can explain how to determine the total tiles by first determining the total difference in tiles of each square shape. Based on the results of the interview and the analysis of the answer to question number 1, it can be concluded that S1 is able to use patterns to determine unknown patterns. In other words, S1 is able to solve problems related to generalization. The same solving strategy is also carried out by S3 and S6, which is to use the difference in each tile shape to determine the number of gray tiles.

Different solving steps are carried out by S5 in solving problem number 1 as presented in Figure 2.
Based on Figure 2, S5 solves the problem by first determining the total tiles on the 6 x 6 and 7 x 7 square shapes by performing a multiplication operation on each square shape. Thus the 6 x 6 square shape obtained a total of 36 tiles and the 7 x 7 square shape obtained a total of 49 tiles. Next S5 determines the number of gray tiles on each 6 x 6 and 7 x 7 square shape by subtracting the total tiles by the number of previously known black tiles. Thus, it is obtained that the number of gray tiles on a 6 x 6 square shape is 20, and a 7 x 7 square shape is 24.

Based on the answers and interview results, all subjects used number patterns to determine the next pattern, although the completion steps used were different. In other words, there are two strategies that the subject does to solve question number 1. The first strategy is to use the difference on each tile shape to determine the number of gray tiles. The second strategy uses multiplication and subtraction operations to determine the number of gray tile shapes and the total tiles. Furthermore, reflective subjects appear more systematic in the completion step. The subject is able to explain the steps of solving the problem in a coherent and systematic manner. Meanwhile, impulsive subject resolution steps tend to use their own mind or use logic.

Abstraction.

Question number 2 is used to investigate students’ ability to abstract, which is the process of using symbols related to mathematical concepts and properties. Based on the test results, all subjects can solve the questions using the right solving steps and get the right answers as well. The answer of subject S1 in solving question number 2 is presented in Figure 3.
Based on Figure 3, S1 solves the problem by first representing the length with \( x + 2 \) and the width of the rectangle with \( x \). Then determine the information asked in the problem, namely determining the area of the rectangle. Using the rectangular area formula, S1 can determine the area of a rectangle i.e. \( x^2 + 2x \). Then S1 supposes the variable \( x \) with 3, and substituting the rectangle area equation obtained \( 15 \text{ cm}^2 \).

Furthermore, the researcher conducted an interview with S1 regarding the answer to question number 2. Excerpts of the interview with S1 are presented as follows.

Researchers : Try to explain how to solve question number 2?
S1 : I was a little confused in answering question 2
Researchers : What do you do first after reading this question 2??
S1 : My first step is to write down what is known and asked in the question. Then, I remembered the rectangular area formula which is length times width. So the length is substituted \( x + 2 \) and the width is \( x \), hence the area of the rectangle \( x^2 + 2x \). After that, I create an example if \( x \) is 3 then the area of the rectangle is \( 15 \text{ cm}^2 \).

Researchers : If the value of \( x = 1 \) then what area of the rectangle?
S1 : Can, \( 3 \text{ cm}^2 \).

Based on the interview results, S1 can explain how to determine the area of a rectangle using the rectangular area formula. Thus, it can be concluded that S1 is able to carry out the abstraction process, which is to use symbols to represent relationships between concepts or mathematical properties. The same strategy is also carried out by all subjects to solve problem number 2, namely by using multiplication operations and rectangular area formulas.
Question number 3 is also an abstraction problem used to investigate the process of using symbols related to mathematical concepts and properties in students. Based on the test results, S3 and S6 are able to complete using the right completion steps and get the right answers as well. S1 was able to use the right completion steps but the answer given was wrong. Meanwhile, S2, S4, and S5 are not able to solve the problem correctly, both the solving steps used and the answers obtained. The answer S3, reflective subject, in solving question number 3 is presented in Figure 4.

Based on Figure 4, S3 solves the problem by first determining the area of the shaded area by subtracting the area of the rectangle area by the area of the unshaded area. Based on the information in the problem, S3 represents the length of a rectangle with \( p = x + 4 \) and rectangular width with \( l = x \). Thus obtained rectangular area i.e. \( x^2 + 4x \) and the area of non-shaded area i.e. \( x \). Then, S3 determines the area of the shaded area using a subtraction operation i.e. the area of the rectangle minus the area of the unshaded area is obtained \( x^2 + 3x \).

Next, the researcher conducted an interview with S3 regarding the answer to question number 3. Excerpts of the interview with S3 are presented as follows.

Researchers : How do you determine the area of shaded area?

S3 : First, I determine the area of the rectangle using the rectangular area formula so that it is obtained \( x^2 + 4x \). Then, to determine the unshaded area, I use the area of the rectangle obtained \( x \). After that, to determine the area of the shaded area by performing a reduction

![Figure 2. S3 Answer to Question Number 3](image-url)
operation between the rectangular area and the area of the
unshaded area is obtained $x^2 + 3x$.

Based on the results of the interview, S3 can explain how to determine the area of the
shaded area by determining the relationship of known symbols in the problem. Thus, it can
be concluded that S3 is able to use symbols related to mathematical concepts and
properties. In other words, S3 is able to show abstraction ability in solving problems.

Then the step of solving S6, the impulsive subject, in solving question number 3 is
presented in Figure 5.

![Figure 5. S6 answer to question number 3](image)

Based on Figure 5, S6 solves the problem directly using the shaded rectangular area
formula. S6 determines the length of the shaded rectangle by subtracting the overall length
by 1, obtained $x + (4 - 1)$. Then, S6 determines the shaded area of the rectangle i.e.
$x^2 + 3x$.

Next, the researcher conducted an interview with S6 regarding the answer to question
number 3. Excerpts of the interview with S6 are presented as follows.

Researchers: How do you determine the area of shaded area?
S6: Just like number 2, which uses the rectangular area formula. First
determine the length of the rectangle, which is $x + (4 - 1)$, obtained
length $(x + 3)$, then substitute the values of length and width into the
rectangular formula so that it is obtained $x^2 + 3x$

Based on the interview results, S6 can explain how to determine the area of a shaded area
using the rectangular area formula. S6 can use symbols to represent relationships between
concepts or mathematical properties. In other words, S6 is able to demonstrate abstraction
ability in solving problems.

The results showed that, reflective and impulsive subjects were able to complete
generalizations appropriately. All subjects were able to use number patterns to determine
or generalize subsequent patterns. However, the strategies used by reflective and impulsive
subjects appear different. Reflective subjects appear to use more systematic resolution
steps. Meanwhile, impulsive subjects tend to put forward logic or use ways of solving according to their own thoughts. Furthermore, in abstraction questions number 2 and 3, the ability of reflective and impulsive subjects tends to vary. In question number 2, all subjects were able to solve abstraction-related problems correctly. In other words, all subjects can use symbols to abstract the relationships between mathematical concepts precisely. The strategies used by reflective and impulsive subjects also tend to be relatively similar.

Furthermore, in question number 3, only 1 person in each cognitive style was able to solve correctly. Meanwhile, other subjects are unable to solve abstraction-related problems correctly. The solving strategies used by reflective and impulsive subjects to solve problem number 3 also appear to be different. As in solving question number 2, reflective subjects use systematic and coherent solving steps. Meanwhile, impulsive subjects tend to use the way of solving according to their own thinking.

Differences in characteristics in solving mathematical problems in students with different cognitive styles are also supported by several studies. Shoimah et al., (2018) explained that reflective subjects tend to be better able to formulate, apply and interpret solutions to geometry material compared to cognitively impulsive style students. Then Satriawan et al., (2018) stated that the reflective subject is able to establish a relationship between the known information and the one asked using a calculation operation. Fajriyah and Asikin (2019) mentioned that reflective subjects are very good at communicating and providing arguments regarding the extent of the solution obtained. Salido et al., (2020) also mentioned that reflective subjects are able to use strategies in finding answers and solving difficult problems using logic through experiments. Further research by Prayitno et al., (2022) found that the reflective subjects solving problems using systematic solving steps, being careful in solving given problems by writing down known information, asking, and making examples of operating equations, and making conclusions.

The characteristics of impulsive subjects who tend to use their own way of thinking more in solving problems are also supported by several studies. Aprilia et al., (2015) stated that impulsive subjects tend to be quick in responding to questions given during the interview and writing down all the ideas that are on their minds during the test. Tabrizi and Iranpour (2015) explained that impulsive subjects tend to be quick in making decisions even though the accuracy of correct answers is relatively low. Then research by Fadiana, (2016)
also explained that impulsive subjects in solving story problems tend to rush so that answers are relatively incorrect and reluctant to review the answers that have been found. Azhil et al., (2017) explained that impulsive subjects are able to solve problems using mathematical expressions, but they tend not to write down formulas that should be used to solve problems. Faradillah (2018) also explained that impulsive subjects tend to rush in analyzing solutions to solve problems. Then Nugroho et al., (2020) Explained that impulsive subjects in solving problems tend to lack understanding of the variables used.

In general, the reflective and impulsive subjects are able to solve generalization and abstraction problems. Thus, reflective and impulsive subjects are able to use patterns to solve problems related to generalization. In addition, reflective and impulsive subjects are able to use symbols as abstractions of the relationship between concepts and mathematical properties. However, in the solution steps, the reflective subject is more systematic and coherent. Meanwhile, impulsive subjects tend to use their own way of thinking to solve problems. Differences in thinking characteristics or cognitive styles need to be facilitated by educators or teachers so that students can be more successful in learning, especially in solving mathematical problems. The difference is supported by Muyassaroh dan Masduki, (2023) which found a relationship between cognitive styles and students' algebraic thinking skills. Therefore, differentiated learning needs to be a consideration in the learning process in the classroom so that students are more successful in learning mathematics.

CONCLUSION

The cognitive style of students exerts an influence on the way students solve math problems. Students with reflective and impulsive cognitive styles are able to demonstrate problem-solving abilities related to generalization and abstraction which are components of algebraic thinking. Reflective and impulsive subjects may demonstrate the ability to use known patterns to generalize subsequent patterns. Reflective and impulsive subjects are also able to use symbols to abstract relationships between mathematical concepts. However, there are differences in resolution steps between reflective and impulsive subjects. Reflective subjects use more systematic resolution steps than impulsive subjects. Impulsive subjects are more likely to use their own logic to solve problems. The implication of the results of this study is that individual differences such as cognitive styles need to be
the attention of teachers by designing appropriate learning strategies so that each individual's differences can be facilitated.

ACKNOWLEDGMENTS
The researcher expressed his gratitude to the students who were willing to participate in the study. The researcher also thanked the school for granting permission to the researcher to take data on students at the school.

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