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RELATIONAL REASONING IN MATHEMATICAL PROBLEM SOLVING: A STUDY OF HIGH SCHOOL STUDENTS WITH IMPULSIVE AND REFLECTIVE COGNITIVE STYLES

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Abstract

Relational reasoning plays an important role in mathematical problem solving because it allows students to logically connect facts, concepts, principles, and operations to obtain meaningful solutions. However, students still tend to rely on memorized procedures without understanding the relationships between mathematical objects, thus experiencing difficulties in planning and justifying problem solving. This study aims to describe the relational reasoning of high school students with impulsive and reflective cognitive styles in solving mathematical problems on the topic of Three Variable Linear Equation Systems (TVLES). The study used a descriptive qualitative approach with a case study design. The research subjects consisted of two eleventh grade students who had average mathematical abilities and represented impulsive and reflective cognitive styles based onMatching Familiar Figures Test (MFFT). Data were collected through problem solving task (PST) based interviews, validated with time triangulation, and analyzed using the analysis model of Miles et al. (2014). The results showed that both subjects were able to build relationships between facts, concepts, principles, and operations at each stage of Polya's problem solving. However, the characteristics of the reasoning process shown were different. Reflective students understood the problem more carefully, considered alternative strategies, and implemented solutions systematically, while impulsive students worked faster, provided more concise justifications, and corrected errors through trial and error. The research findings indicate that cognitive style influences the characteristics of the relational reasoning process in building and explaining relationships between mathematical objects at each stage of problem solving. The results of this study provide an overview of the characteristics of students' relational reasoning that can be used as a basis for designing mathematics learning that is more appropriate to students' cognitive characteristics.

Keywords: Cognitive Style, Impulsive, Mathematical Problem Solving, Reflective, Relational Reasoning

Abstrak

Penalaran relasional berperan penting dalam pemecahan masalah matematis karena memungkinkan siswa membangun hubungan logis antara fakta, konsep, prinsip, dan operasi matematika. Namun, siswa masih cenderung mengandalkan prosedur yang dihafal tanpa memahami hubungan antarobjek matematika sehingga mengalami kesulitan dalam merencanakan dan memberikan justifikasi terhadap penyelesaian masalah. Penelitian ini bertujuan mendeskripsikan penalaran relasional siswa SMA bergaya kognitif impulsif dan reflektif dalam pemecahan masalah matematis pada materi Sistem Persamaan Linear Tiga Variabel (SPLTV). Penelitian ini menggunakan pendekatan kualitatif deskriptif dengan desain studi kasus yang melibatkan dua siswa kelas XI berkemampuan matematika sedang yang mewakili gaya kognitif impulsif dan reflektif berdasarkan *Matching Familiar Figures Test* (MFFT). Data dikumpulkan melalui wawancara berbasis tugas pemecahan masalah (TPM), divalidasi dengan triangulasi waktu, dan dianalisis menggunakan model Miles et al. (2014). Hasil penelitian menunjukkan bahwa kedua subjek mampu membangun relasi antara fakta, konsep, prinsip, dan operasi pada setiap tahap pemecahan masalah Polya, tetapi menunjukkan karakteristik penalaran yang berbeda. Siswa reflektif memahami masalah secara lebih cermat, mempertimbangkan alternatif strategi, serta menyelesaikan masalah secara sistematis, sedangkan siswa impulsif bekerja lebih cepat, memberikan justifikasi yang lebih ringkas, dan memperbaiki kesalahan melalui proses coba-coba. Temuan ini menunjukkan bahwa gaya kognitif memengaruhi karakteristik penalaran relasional dalam membangun dan menjelaskan relasi antarobjek matematika pada setiap tahap pemecahan masalah serta memperkaya pemahaman mengenai penalaran relasional siswa sebagai acuan dalam pengembangan pembelajaran matematika yang lebih sesuai dengan karakteristik kognitif siswa.

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Kata kunci: Gaya Kognitif, Impulsif, Pemecahan Masalah Matematis, Penalaran Relasional, Reflektif

INTRODUCTION

Mathematics plays a crucial role in developing the logical, analytical, and systematic thinking skills students need to solve various problems (Duma et al., 2024). Therefore, at the high school level, mathematics learning is directed not only at mastering calculation procedures but also at developing mathematical thinking skills as a basis for problem-solving. Niss and Højgaard (2019) argued that mathematical reasoning is one of the skills that support this process, as it enables students to understand the rationale behind a procedure, explain relationships between concepts, and draw logical conclusions in solving problems.

However, mathematics learning in schools still faces challenges in developing students' reasoning skills. This occurs because students tend to rely on memorized procedures without understanding the relationships between concepts, even though mathematics is fundamentally a system of knowledge built on the relationships between various concepts (Mukuka et al., 2023; NCTM, 2000). As a result, students experience difficulties when faced with problems that require deeper reasoning, particularly in determining solution strategies and justifying the steps taken (Chusnudhin et al., 2023; Gultom et al., 2022).

Students' difficulties in connecting mathematical information and concepts underscore the importance of a skill that enables them to logically establish relationships between facts, concepts, principles, and operations during the problem-solving process. Alexander et al. (2022) explain that relational reasoning involves a series of cognitive processes that enable individuals to construct and utilize relationships between pieces of information. Similarly, Cortes et al. (2021) state that relational reasoning focuses not only on understanding objects individually, but also on the ability to evaluate relationships between mental representations, so that students can understand the interrelationships between mathematical elements as a whole. Relational reasoning is therefore an essential skill in mathematical problem-solving, as it helps students build meaningful relationships as a basis for determining solution strategies and justifying each step taken.

Relational reasoning is defined as the process of recognizing meaningful patterns and establishing logically justified conceptual relationships (Halford et al., 2010). In the context of mathematics, this ability is reflected when students connect facts, concepts, principles, and operations to build a comprehensive understanding (Cortes et al., 2021). Based on this perspective, the present study defines relational reasoning as the process of relating mathematical objects and providing logical justification in solving mathematical problems.

Mathematical problem-solving is viewed as a process that requires students to utilize various mathematical ideas, knowledge, and skills in an integrated manner (Saputri and Ananda, 2026). In line with this view, Molina and Castro (2021) argued that relational reasoning in this process is demonstrated not only through the ability to apply procedures or formulas, but also through the ability to explain the reasons underlying their use and to understand the mathematical structure of the problem. Therefore, Polya (1973) problem-solving stages understanding the problem, devising a plan, carrying out the plan, and looking back can therefore be used as a framework to identify how relational reasoning is constructed at each stage of problem-solving. At each stage, students construct, use, and evaluate relationships between mathematical objects, including facts, concepts, principles, and operations, as a basis for arriving at an appropriate solution..

Differences in the characteristics of students' relational reasoning are thought to be influenced by individual differences in processing information, commonly referred to as cognitive style. Majidi et al. (2025) stated that individuals differ in how they process information and organize their thinking, resulting in variations in how they structure information and approach learning experiences (Prawita et al., 2022). In mathematics learning, one dimension of cognitive style that is widely studied is impulsive and reflective. Students with an impulsive cognitive style tend to respond quickly without in-depth consideration, while reflective students tend to understand problems more carefully and consider several alternative solutions before providing an answer (Faisal et al., 2023; Patta et al., 2021; Sholikin, 2025; Susiloningsih et al., 2024). These differences in characteristics are thought to influence students' relational reasoning processes in solving mathematical problems.

Previous studies have examined relational reasoning in relation to various individual characteristics, such as personality type (Cahyani, 2025; Susanti et al., 2025). Research on cognitive style, meanwhile, has focused more on analogical reasoning (Fitroni & Rosyidi, 2025), mathematical problem-solving in general (Aisy et al., 2021), and numeracy ability (Simamora & Akhiruddin, 2022). These findings indicate that individual characteristics contribute meaningfully to students' mathematical thinking processes, yet their specific relationship to relational reasoning remains underexplored.

To date, no study has been found that specifically examines how students with impulsive and reflective cognitive styles construct relational reasoning at each stage of Polya's problem-solving process. Addressing this gap is important for explaining the characteristics of the relationships between facts, concepts, principles, and operations that students construct during problem-solving, thereby providing a stronger empirical basis for designing instruction that accommodates students' cognitive characteristics.

Based on this gap, the present study aims to describe the characteristics of relational reasoning among high school students with impulsive and reflective cognitive styles at each stage of Polya's problem-solving process in solving three-variable linear equation system (TVLES) problems. This study is expected to contribute to the literature on relational reasoning in mathematics education and to provide practical insight for teachers in designing instruction that accommodates students' cognitive characteristics to support their mathematical problem-solving abilities.

METHODS

Research Design and Subjects

This research employed a descriptive qualitative approach with a case study design, aimed at describing the relational reasoning of senior high school students in solving TVLES problems. The design was chosen because relational reasoning is a cognitive process that is best captured through in-depth, naturalistic exploration rather than numerical measurement alone. Two cognitive styles were compared within this design, namely reflective and impulsive, to examine how each style shapes the way students connect mathematical facts, concepts, principles, and operations during problem solving.

Subjects were selected through a staged criterion-based sampling procedure from class XI C of MAN 1 Tuban. From the initial population, female students with moderate mathematical ability were first identified based on report-card scores and teacher recommendation, after which the MFFT was administered to determine each student's cognitive style tendency. Two subjects with the highest reflective and impulsive tendencies were ultimately selected, with only a one-point difference in their mathematics scores, so that differences in relational reasoning could be attributed to cognitive style rather than to gender or ability level.

Research Instruments

The primary instruments used in this study were the MFFT, two equivalent Problem-Solving Tasks (PST 1 and PST 2) on TVLES material, and a semi-structured interview guide. Each PST consisted of one descriptive item with two related questions, designed to elicit how subjects related given information to the steps of Polya (1973) problem-solving stages. Both instruments were developed as parallel forms so that data obtained from PST 1 could later be cross-checked against PST 2 for consistency.

Prior to use, the PST instruments underwent expert validation by a mathematics education lecturer and content confirmation by the class's mathematics teacher, followed by a readability test involving three non-subject students. The validation process led to minor revisions, such as replacing the generic phrase "another student" with a named character to avoid ambiguity, while the readability test identified one sentence that was initially difficult for students to interpret and was subsequently clarified. These revisions were made without altering the mathematical structure or the measurement objective of the tasks, so the final instruments were considered appropriate for eliciting subjects' relational reasoning.

Data Collection Procedure

Data were collected through task-based interviews, in which each subject worked on a PST while thinking aloud and was subsequently probed by the researcher following Polya's four problem-solving stages. Each subject completed both PST 1 and PST 2 in separate interview sessions, with the second task serving as a means of verifying the consistency of the reasoning patterns identified in the first. All interviews were audio-recorded and transcribed verbatim to preserve the exact wording used by the subjects in explaining their reasoning.

Data validity was established through time triangulation, comparing the relational reasoning patterns that emerged from the PST 1 and PST 2 interviews for each subject. Data in which the two tasks showed consistent patterns were considered valid and retained for further analysis. To facilitate systematic identification, each transcript excerpt was coded using a six-digit system distinguishing the interviewer and subject, cognitive style, interview order, and item sequence.

Data Analysis

Data were analyzed using the interactive analysis model of Miles et al. (2014), comprising data condensation, data display, and conclusion drawing/verification. Analysis was carried out based on relational reasoning indicators adapted from Halford et al. (2010), focusing on how facts, concepts, principles, and operations were connected and accompanied by logical justification at each stage of Polya (1973) framework. Coded relations were then compiled into indicator tables to allow systematic comparison between the reflective and impulsive subjects.

To support analytical rigor, each identified relation was evaluated on two aspects: the correctness of the relation formed and the logical adequacy of the reasoning given by the subject. This dual evaluation allowed the researcher to distinguish between subjects who merely arrived at correct answers and those who demonstrated genuine relational understanding through well-justified connections. The resulting analysis formed the basis for describing and comparing the relational reasoning profiles of the reflective and impulsive subjects presented in the Results and Discussion section.

RESULTS AND DISCUSSION

Based on the results of the subject selection described in the Methods section, two female students of class XI C were selected with relatively equal, moderate mathematical ability: Subject 1 with the highest tendency toward a reflective cognitive style, and Subject 2 with the highest tendency toward an impulsive cognitive style. Because their mathematical abilities were nearly equal, the differences in relational reasoning that emerged can be attributed more to differences in cognitive style than to differences in mathematical ability.

The relational reasoning analysis in this section is based on the relationships between mathematical objects, namely facts (F), concepts (K), principles (P), and operations (O), which are numbered sequentially according to their order of appearance during the problem-solving process; for example, F4 refers to the fourth fact and P2 to the second principle identified in the data. Interview excerpts are marked with a six-digit code; for instance, SR1-030 indicates the transcript of the reflective subject (SR) in the first interview (1), dialogue sequence 30, while the code SI refers to the impulsive subject. The validity of the data at each Polya stage was established through time triangulation, comparing the consistency of subjects' answers and explanations across PST 1 and PST 2; all data presented in the following section have been declared valid based on this triangulation.

RESULTS

Subject 1 Relational Reasoning (Reflective) in Solving Mathematical Problems

The interview data based on PST of Subject 1 solving TVLES problems were analyzed by first identifying the mathematical objects that emerged during the subject's thinking process, then grouping them into four categories: facts (F), concepts (K), principles (P) and operations (O). This grouping was done repeatedly by referring to the task-based interview transcript, so that each resulting code could be traced back to the subject's specific statement. The complete definition of each mathematical object code identified in Subject 1 is presented in Table 1.

Table 1. Classification of Subject 1 Mathematics Object

Polya Stage	Code	Short Meaning
Understanding the Problem	F1	Subject name (Rina, Dodi, Siska, Lina)
	F2	Type & quantity of items purchased by each student
	F3	Total known shopping price
	F4	Rina's total purchases = price of 1 pencil case
	F5	Unit price of each item (value asked)
	O1	Add up the prices per item for Lina's total purchases.
Making a Plan	K1	Substitution method as a solution strategy
	K2	Variable example: book = b, pencil = p, pencil case = t
	P1	The same values can replace each other
	P2	Direct elimination is only possible if the variables are equivalent.
	P3	Translation result equation (Rina, Dodi, Siska)
	O2	Translating story sentences into algebraic equations
Executing the Plan	O3	Substitute Rina's equation into Dodi's equation
	P4	New equation resulting from substitution
	O4	Find the difference between the substitution results using the Siska equation
	F6	Pencil price = Rp. 6,000
	O5	Substitute the price of pencils into Siska's equation
	F7	Book price = Rp. 10,000
	O6	Substitute the price of books & pencils into Rina's equation
	F8	Pencil case price = Rp. 22,000
	O7	Substitute into Lina's equation
	F9	Lina's total spending = Rp. 60,000
	P5	The order of completion follows the relationship of the variables.
Looking back	O8	Back substitution for verification
	O9	Interpretation of results into the contextual meaning of the question
	P6	Rina's substitution must be done first
	P7	The solution is single-minded

Based on the grouping of mathematical objects, the relationships between objects constructed by Subject 1 were analyzed to see how facts, concepts, principles, and operations are interconnected at each stage of problem solving. The results of this relationship analysis are presented in Table 2, which shows the sequence of relationship formation from the problem understanding stage to the review stage.

Table 3. Subject 1 Relational Reasoning in Solving Mathematical Problems

Polya Stage	Mathematical Object Relations	Description
Understanding the Problem	1) F1–F2 2) F2–F3 3) F2–F4 4) F5–O1	Linking student identity, item details, total price, and equivalence facts accurately and in detail, justified directly by the numbers and explicit information in the question. The habit of reading questions repeatedly before answering reflects a careful, reflective character.
Making a Plan	1) F4–K1 2) P2–K1 3) P1–P3 4) F2/F3–K2/P3 (via O2)	Establishing a substitution strategy after considering alternative methods, with logical and coherent justification. Reflectiveness is evident in a willingness to consider options before making a decision, rather than immediately choosing the first strategy that comes to mind.
Executing the Plan	1) P3–O3 2) P4–O4 3) F6–O5 4) F6/F7–O6 5) F6/F7/F8–O7 6) P5–O3/O4/O5/ O6/O7	Continuously solve 6 relations (substitution–elimination–continuation substitution) without error; each result is justified as the basis for the next step. Reflective character is evident from systematic execution and minimal revision, the result of careful planning.
Looking back	1) F6/F7/F8–O8 2) F11–O9 3) K1/O3–P6 4) O8–P7	Verifying results against initial facts, translating them into the context of the problem, and concluding the uniqueness of the solution with detailed structural justification. Reflectiveness is evident in a thorough evaluation of the process and strategy, not simply ensuring the correct answer.

In the problem understanding stage, Subject 1 read the problem for a relatively long time before explaining his understanding. Subject 1 connected the student's identity with the details of his purchases (F1–F2), as seen when explaining that "Dodi bought 2 books, 1 pencil, and 1 pencil case" and that the numbers "indicate the number of items purchased... which describes the subject, namely Dodi" (SR1-002, SR1-006). Subject 1 also connected the item details with the known total price (F2–F3), and identified the equivalence fact formed from the information in the problem (F2–F4). Next, Subject 1 connected the unit price asked with the steps to be taken in solving (F5–O1): "to find Lina's total purchases, we must first find the price per item" (SR1-008).

In the planning stage, Subject 1 connected the fact of equivalence with the choice of solution strategy (F4–K1): the substitution strategy was chosen "because if you look at the total of Rina's, it is equal to 1 pencil case, so it can be directly substituted for Dodi" (SR1-010). Subject 1 also considered other methods before making a choice (P2–K1): direct elimination was stated as "not possible" "because the equation has 2 and 3 variables" (SR1-011). Subject 1 then connected the general substitution principle with its application to the equation of the translation problem (P1–P3): "if there are two things that have the same value, we can substitute each other... 1 pencil case can be replaced with 1 book plus 2 pencils" (SR1-012). The details of the items and the total price that had been identified in the previous stage were also connected with the example of variables and the equation of the translation of the story sentence (F2/F3–K2/P3, through the translation process O2).

In the planning implementation stage, Subject 1 performed substitution (P3–O3), applied elimination after equating the equations (P4–O4), and used the result of one step to determine the next variable (F6–O5; F6/F7–O6; F6/F7/F8–O7). This sequence of operations is connected by the principle that the order of solution follows the relationship between variables (P5–O3/O4/O5/O6/O7), as explained by Subject 1: "we have to substitute Rina's first for Dodi's so that the pencil case variable disappears... if the order is randomized, it will not be possible to solve" (SR1-024). The results of Subject 1's written work at this stage are presented in Figure 1.

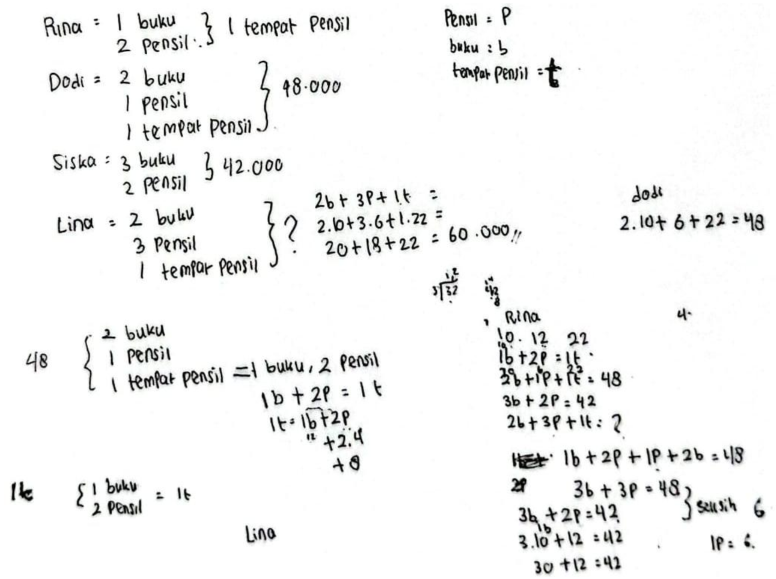

Figure 1. Written Work Results for Subject 1

In the review stage, Subject 1 reconnected the obtained values to known facts (F6/F7/F8–O8): "I tried to accumulate everything... and the result was the same as the total expenditure, which was Rp. 48,000" (SR1-027). The final value obtained was reconnected to the contextual meaning of the problem, namely Lina's total expenditure which was asked since the problem understanding stage (F11–O9). Subject 1 also connected the steps that had been taken with the understanding of the order that must be maintained (K1/O3–P6): "I still have to go through Rina's substitution step first, sis, because Rina's doesn't have a number" (SR1-029), and concluded the uniqueness of the solution (O8–P7): "from the difference between Dodi and Siska, the price of the pencil must be Rp. 6,000... if one of the prices is changed, it definitely won't match" (SR1-031).

Overall, the relationships built by Subject 1 developed from fact-based relationships at the problem-understanding stage, to conceptual and principle-based relationships at the planning stage, followed by a series of operations at the implementing stage, and back to evaluative relationships at the review stage. In each of these relationships, Subject 1 accompanied logical justifications explaining why the objects were related, for example, explaining that the substitution was chosen because of the equivalence of value between Rina's total spending and the price of a pencil case (F4–K1), explaining the conditions for a method to be used before deciding not to use it (P2–K1), and explaining the sequence of work based on the relationship between variables, not just a memorized sequence. Thus, both indicators of relational reasoning in this study in relating mathematical objects and providing logical justification for the relationship were fulfilled by Subject 1 consistently throughout all stages of Polya's problem-solving, with each relationship accompanied by reasons referring to the numerical structure or equivalence of values in the problem, not simply procedures carried out without explanation.

Subject 2 Relational Reasoning (Impulsive) in Solving Mathematical Problems

The interview data based on PST of Subject 2 in solving SPLTV problems were analyzed by grouping the same mathematical objects as Subject 1, namely into the categories of facts (F), concepts (K), principles (P), and operations (O). The complete definition of each mathematical object code identified in Subject 2 is presented in Table 3.

Table 3. Classification of Mathematical Objects for Subject

Polya Stage	Code	Short Meaning
Understanding the Problem	F1	Student names: Rina, Dodi, Siska, Lina
	F2	Type & quantity of items purchased by each student
	F3	Total known shopping price
	F4	Known relationship: 1 book + 2 pencils (Rina) = price of 1 pencil case
	F5	Unit price of the item (asked)
	O1	Use of unit price to calculate Lina's total expenses
Making a Plan	K1	Substitution and elimination methods as strategies
	K2	Variable example: book = x, pencil = y, pencil case = z
	P1	The same values can replace each other

Polya Stage	Code	Short Meaning
	P2	Rina's equation has no total value, substitution must come first
	O2	Translate story sentences into mathematical symbols
Executing the Plan	O3	Writing the equations of Rina, Dodi, Siska in symbolic form
	P3	Translation result equation
	O4	Initial attempt at direct elimination of equations 2 & 3 (incorrect)
	P4	Direct elimination is only possible if the variables are equivalent.
	O5	Substitute the 1st equation into the 2nd equation
	P5	New equation resulting from substitution
	O6	Eliminate the substitution result with the 3rd equation
	F6	Pencil price = Rp. 6,000
	O7	Attempt to substitute y into the 2nd equation (wrong)
	O8	Substitute y into the 3rd equation (after fixing)
	F7	Book price = Rp. 10,000
	O9	Substitute x, y into the 2nd equation
	F8	Pencil case price = Rp. 22,000
	O10	Substitute x, y, z into the Lina equation
	F9	Lina's total spending = Rp. 60,000
	P6	The order of completion follows the relationship of the variables.
Looking back	O11	Back substitution for verification into Dodi's equation
	O12	Interpretation of results into the contextual meaning of the question
	P8	Rina's substitution must be done first (recurring)
	P9	The solution is single-minded

Based on the grouping of mathematical objects, the relationships between objects constructed by Subject 2 were analyzed to see how facts, concepts, principles, and operations were interconnected at each stage of problem-solving. The results of this relationship analysis are presented in Table 4.

Table 4. Subject 2 Relational Reasoning in Solving Mathematical Problems

Polya Stage	Mathematical Object Relations	Description
Understanding the Problem	1) F1–F2 2) F2–F3 3) F2–F4 4) F5–O1	Linking the same elements as Subject 1, but the justification was brief and there was some inconsistency between the verbal explanation and the item composition. Impulsive character is evident from the quick response without carefully rereading the questions.
Making a Plan	1) F2/F3–K1 2) F4–P2 3) P1–F4 4) K2–O2	Immediate substitution strategy adoption without considering alternatives; relationships remain valid, but justifications tend to be brief and lacking depth. Impulsiveness is evident in the speed with which strategic decisions are made without a comparative process.
Executing the Plan	1) O3–P3 2) O4–P4 3) P3–O5 4) P5–O6 5) F6–O7/O8 6) F6/F7–O9 7) F6/F7/F8–O10 8) P6–O5/O6/O7/O8 /O9/O10	Completed 8 relationships; 2 of which were initially flawed (try-then-fix) before realizing and correcting them on their own. His impulsive nature is evident in his hasty initial execution, but is balanced by his strong self-correction skills, resulting in a correct final result.

Polya Stage	Mathematical Object Relations	Description
Looking back	1) F6/F7/F8–O11 2) F9–O12 3) K1/O5–P6 4) O11–P7	Verification and contextual linking of the problem were achieved, but justification was shorter and shallower. Impulsiveness remained, but spontaneous reflection on early mistakes emerged, indicating metacognitive awareness even though it was not requested.

At the understanding stage, Subject 2 responded to questions relatively quickly and immediately explained his understanding without a long pause. Subject 2 connected the student's identity with the details of his purchases (F1–F2): "Dodi bought 2 books, 1 pencil, and 1 pencil case for Rp48,000" (SI1-002), but the accompanying reasoning was still brief, namely that it was "already mentioned in the problem so it is known information" (SI1-006). Subject 2 was also less careful when connecting the details of the items with the total price (F2–F3): cross-checking at the implementing the plan stage showed that the composition of Siska's purchases as stated verbally (2 books, 3 pencils) was inconsistent with the composition actually used in the calculation (3 books, 2 pencils), but this discrepancy was not recognized by Subject 2 at this stage. Subject 2 still identified the fact of equivalence (F2–F4) and connected the unit price with the settlement plan (F5–O1): "Lina bought the same item, sis, so the known price can be used to find Lina's total spending" (SI1-009).

In the planning stage, Subject 2 connected the fact of the three students' purchases with the choice of strategy (F2/F3–K1), with a less firm level of confidence: "because from the purchases of these 3 students, we can make an equation so maybe it can be substituted and eliminated" (SI1-011). Subject 2 connected the fact that Rina did not have a total value with the need to make a substitution first (F4–P2): "because Rina does not have a total price, sis, there is only an equivalent pencil case" (SI1-012). The principle that equal values can be substituted for each other also became the basis for the choice of the strategy (P1–F4), although consideration of other methods was only answered briefly without further exploration: "maybe there is, but I use substitution and elimination" (SI1-016). Next, the example of variables was connected with the process of translating the story sentence into symbolic form (K2–O2).

In the implementation stage of the plan, Subject 2's implementation of the plan was marked by two experimental steps that were initially wrong and were later realized and corrected independently (O4–P4; F6–O7/O8). In the first experiment, Subject 2 tried to directly eliminate two equations with disproportionate variables: "Initially, I wanted to directly eliminate equations 2 and 3, but after I calculated, it seems my method was wrong, I didn't eliminate them first. Because of that, the variables were different, there were 3 and 2, so I crossed them out" (SI1-019). The relationship between the translation equation and the first substitution step (O3–P3; P3–O5), as well as the new equation resulting from the substitution with the next elimination step (P5–O6), was also formed in this series. After correcting the wrong steps, Subject 2 was still able to summarize the entire solution sequence in its entirety (F6/F7–O9; F6/F7/F8–O10; P6–O5/O6/O7/O8/O9/O10): "because we have to substitute first so that the variable z disappears, then we can eliminate it to find y, sis. Otherwise, there are still 3 variables" (SI1-027). The results of Subject 2's written work at this stage, including the correction process, are presented in Figure 2.

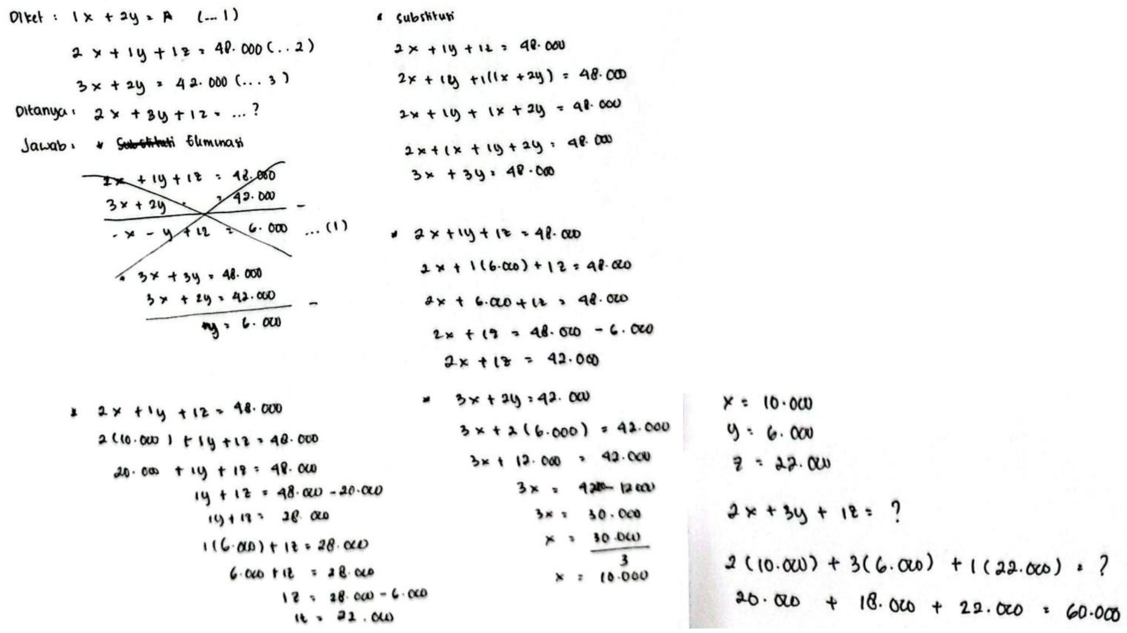

Figure 2. Written Work Results for Subject 2

In the review stage, Subject 2 verified by substituting the obtained values back into one of the equations (F6/F7/F8–O11): "I entered the values of x, y, z into Dodi's equation, sis. 2 times 10,000 plus 6,000 plus 22,000, the result is 48,000, which matches Dodi's total in the question" (SI1-030). The final value obtained was also reconnected with the contextual meaning of the question (F9–O12). Subject 2 understood that the substitution of Rina to Dodi must still be done first, although the order after that can vary (K1/O5–P6): "you still have to substitute Rina to Dodi first. The order might be different... because Rina doesn't have a total price, sis" (SI1-033, SI1-034), and believed in the uniqueness of the solution based on the results of the check (O11–P7): "I've checked and the result matches the total in the problem, sis" (SI1-036). Beyond the mathematical object code, Subject 2 also showed spontaneous metacognitive awareness of errors that occurred at the beginning of the work: "maybe there was a wrong step at the beginning. I think it was a wasted step, I should have thought about it first" (SI1-037).

Overall, the relationships built by Subject 2 developed from fact-based relationships at the problem-understanding stage, to conceptual and principle relationships at the planning stage, followed by a series of operations at the plan-implementing stage and returning to evaluative relationships at the review stage. In each of these relationships, Subject 2 continued to provide reasons connecting the mathematical objects, for example, explaining the basis for selecting a strategy based on the relationship between the three students' purchases (F2/F3–K1), or explaining the elimination conditions after realizing an error in the first experiment (O4–P4) even though some of these justifications were more concise than Subject 1's. Thus, the two indicators of relational reasoning in this study relating mathematical objects and providing logical justification for the relationship were also fulfilled by Subject 2 at all stages of Polya's problem-solving, including the relationships that had experienced errors before being corrected by themselves.

DISCUSSION

Subject 1's relational reasoning demonstrated a gradual development consistent with Halford et al.'s (2010) concept of relational complexity: from fact-based relationships to concept-principle relationships, evolving into interrelated operational relationships, and finally, evaluative relationships. This structure began to form at the problem-understanding stage, when Subject 1 linked unit prices to solution steps (F5–O1) as a form of anticipation. According to Alexander et al. (2022), this ability is central to relational reasoning because it demonstrates anticipation of the solution structure before a plan is formulated. This anticipatory pattern can be explained by the tendency of reflective individuals to allocate more cognitive resources to the information encoding stage before responding (Faisal et al., 2023); because more relationships are evaluated early on, errors occur in the implementation stage, which also explains why Subject 1 was able to complete all relationships at the plan implementation stage without revision. For each relationship constructed, indicators of logical justification relational reasoning were also evident in Subject 1's willingness to consider alternative methods before deciding on a strategy (P2–K1), in line with the views of Cortes et al. (2021) that relational reasoning also includes evaluating the relationships between representations, not just the direct application of a single procedure.

In contrast to Subject 1, Subject 2 demonstrated a relational reasoning pattern that was structurally similar but proceeded through a different process. At the problem-understanding stage, the relationships established were equivalent in type and quantity to Subject 1, but the justifications provided tended to be concise and direct without elaboration. There was even an unconscious discrepancy (F2–F3) between the verbally stated expenditure composition and that used in the calculation. This indicates a tendency to respond impulsively without thorough examination (Shabshai, 2020). This pattern continued into the planning stage, where consideration of alternative strategies was not explored in more depth. Interestingly, at the plan-executing stage, Subject 2 demonstrated two stages of awareness and self-correction of his own errors (O4–P4). From the perspective of Cortes et al. (2021), this condition still reflects an evaluation of the relationships between representations, but it is reactive, occurring after the error occurs, not before the action. This pattern can be understood as a difference in the timing of relational evaluation, not its absence: impulsive individuals appear to prioritize quick initial responses over thorough relational examination, shifting the evaluation process from the encoding stage to the implementation stage. Consequently, errors that would have been caught in the planning stage in Subject 2 emerged during implementation and had to be corrected immediately. In the retrospective stage, metacognitive awareness also emerged that initial steps should have been considered first but were only explicitly realized after the entire process was complete, indicating that evaluation still occurred, only delayed until the end of the process.

Based on these findings, both subjects demonstrated identical relational development directions throughout the Polya stages: fact-based relations at the problem-understanding stage, conceptual and principle relations at the planning stage, a series of operational relations at the plan-executing stage, and evaluative relations at the reviewing stage. This similarity in pattern indicates that relational development follows the structural demands of the Polya stages, rather than being solely determined by cognitive style. Both subjects also completed all stages with correct relations and accompanied by logical reasoning, without skipping a single stage. In other words, both subjects fulfilled both relational reasoning indicators in this study equally. The differences that emerged were more apparent in the quality of the thinking process and the depth of explanation, as Subject 1's relational reasoning completed six relations without error with more detailed justifications for each step, while Subject 2 constructed eight relations through a trial and error process, with justifications that tended to be more concise but still logical. The differences that emerged were more apparent in the quality of the thinking process and the depth of explanation, as summarized in Table 5.

Table 5. Comparison of Relational Reasoning of Subject 1 and Subject 2

Polya Stage	Similarities	Difference
Understanding the Problem	The number and type of relations are identical (three fact–fact relations, one fact–operation relation)	Subject 1 reads longer and more carefully; Subject 2 reads shorter and sometimes less carefully.
Making a Plan	The objects that are related are the same (facts, concepts, principles), each of the four relations	Subject 1 considered alternative methods before making a choice; Subject 2 immediately determined the strategy without the consideration stage.
Executing the Plan	The principle-operation relationship dominates and is built continuously.	Subject 1 completed six relations without error; Subject 2 constructed eight relations through a trial-and-correction process.
Looking back	Number and type of identical relations (F–O, F–O, K/O–P, O–P)	Subject 1 provided a comprehensive structural explanation of the uniqueness of the solution; Subject 2 provided a more superficial reasoning but accompanied by spontaneous metacognitive reflection.

These findings clarify how cognitive style shapes the two relational reasoning indicators underlying this study. In terms of connecting mathematical objects, both subjects showed an equivalent direction of relational development, indicating that this ability can be achieved equally by reflective and impulsive students of comparable mathematical ability. Cognitive style instead shaped the logical justification indicator: Subject 1 accompanied each relation with a more detailed, anticipatory explanation, while Subject 2 offered more concise justification, some of which emerged only after an error occurred. This suggests that students' difficulties with relational reasoning, as outlined in the Introduction, are not solely attributable to cognitive style, but rather to how students process and justify the relations they construct. Thus, reflective and impulsive cognitive styles do not determine students' ability to construct relations between mathematical objects, but instead shape how logical justification is formed, both in depth of explanation and timing of evaluation.

These findings align with Aisy et al. (2021) and Aishah et al. (2024), who reported that reflective students tend to be more careful, and extend Simamora and Akhiruddin's (2022) finding that reflective students work slower but more accurately while impulsive students work faster with errors corrected later. However, these studies generally treat speed, accuracy, and correction as outcome-level differences. This study contributes more specifically by showing that trial-and-error in impulsive students does not change the final structure of the relations formed, only the path to achieving it — impulsivity affects how, not whether, the correct relational structure is reached. Unlike Fitroni and Rosyidi (2025), who examined analogical reasoning in open-ended problems, and Susanti et al. (2025) and Cahyani (2025), who examined relational reasoning through a personality lens, this study offers a more detailed picture of how this pattern unfolds across each stage of Polya's framework, specifically in the context of solving SPLTV.

CONCLUSION

This study concludes that high school students with both reflective and impulsive cognitive styles are equally capable of constructing relational reasoning at all stages of Polya's problem-solving in solving three-variable linear equation systems, with identical relational development directions starting from fact-based relations, moving towards conceptual and principle relations, developing into a series of operational relations, and returning to evaluative relations. In relations between mathematical objects, reflective and impulsive students show equivalent achievements. Conversely, in providing logical justification, cognitive style plays a significant role: reflective students tend to construct relations more carefully with more detailed justification at each stage, while impulsive students work more quickly and occasionally correct steps through trial and error, without changing the accuracy of the relationship structure that is finally formed.

This study has several limitations that need to be considered in interpreting the above findings. The research subjects were limited to two female students with average mathematical abilities in one topic, namely SPLTV. Therefore, the relational reasoning characteristics found may not necessarily apply to other mathematical ability categories, other mathematical topics, or male students with similar cognitive styles. Furthermore, as a qualitative research with a case study design, the depth of analysis obtained implies a limited scope of subjects, so the findings are contextual to the participants and the research setting and cannot be broadly generalized.

Based on these limitations, further research is recommended to examine the relational reasoning of students with reflective and impulsive cognitive styles in the high and low mathematics ability categories, other mathematics topics, and by involving male subjects, to strengthen the applicability of the findings regarding the influence of these cognitive styles. Further research can also explore learning designs that explicitly accommodate the characteristics of both cognitive styles, for example through scaffolding that encourages impulsive students to deepen their justification before acting, as a practical follow-up of the findings of this study for the development of mathematics learning in schools.

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